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BENJAMIN FRANKLIN AND AERONAUTICS.

BY

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On the 27th of August of 1783, Paris witnessed its first balloon ascension. Some two months earlier the Montgolfier brothers, Joseph-Michel and Jacques-Etienne, had startled the world by demonstrating that a balloon could actually be sent up into the air. These brothers had begun their experiments some time toward the end of 1782. They had inflated small bags by holding them over a fire and found that they rose up into the air due, as they incorrectly thought, to the "smoke" or to some "vapour" or other which was emitted by the flame. At Annonay, on June 5, 1783, they gave a first public demonstration of their invention which was also the first test on a large scale. This time the brothers used a linen glove of about 105 feet in circumference which they inflated by holding the mouth over a fire fed with small bundles of straw. When fully inflated, the balloon was released and rose rapidly into the air until, after about ten minutes, the air inside the bag cooled, whereupon it fell, landing on the ground at a point about one mile and a half away from the place of ascent.

Not to be outdone by the provinces, Paris soon had an ascent of its own, organized by Franklin's friend, B. Faujas de Saint-Fond. The balloon was constructed under the di-

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

rection of the distinguished French physicist, J.-A.-C. Charles, and was based upon a new principle, quite different from that used in the Montgolfiers' balloon. Charles had decided that the balloon rose in the air because its specific gravity was less than that of the air, and thought that a more efficient way of obtaining the lower specific gravity was possible.

When ordinary air is heated, he knew, it expands and becomes lighter. But there are many gases which, under ordinary conditions, are lighter than ordinary air; why not try one of them? Thus he decided to use hydrogen gas which was then called "inflammable air." He obtained his supply of hydrogen by the familiar process of the action of dilute sulphuric acid upon iron filings. The hydrogen liberated in this way was conducted into the bag by means of lead pipes.

The bag itself was accurately described by Franklin in a letter to Sir Joseph Banks, the President of the Royal Society of London, in the following words:

> A hollow Globe 12 feet diameter was formed of what is called in England Oiled Silk, here Taffetas *gommée*, the Silk being impregnated with a Solution of Gum-elastic in Lintseed Oil, as is said. The Parts were sewed together while wet with the Gum, and some of it was afterwards passed over the Seams, to render it as tight as possible.[1]

To inflate this bag fully, no less than one thousand pounds of sulphuric acid and five hundred pounds of iron filings was necessary. The process required about three and one half days for its completion. Then, at five o'clock in the afternoon on August 27, 1783, the balloon was liberated from the Champ de Mars and rose with great rapidity to a height of some three thousand feet. It remained aloft for about three quarters of an hour and then fell to the ground about fifteen miles away from the starting point. Franklin described the ascent as follows:

> At 5 o Clock Notice was given to the Spectators by the Firing of two Cannon, that the Cord was about to be cut.

[1] Franklin to Sir Joseph Banks, Passy, Aug. 30, 1783. A. H. Smyth (editor): "The Writings of Benjamin Franklin," 10 vols. (New York, The Macmillan Co., 1907–1910), vol. ix, pp. 79–81.

> And presently the Globe was seen to rise, and that as fast as a Body of 12 feet diameter with a force only of 39 pounds, could be suppos'd to move the resisting Air out of its Way. There was some Wind, but not very strong. A little Rain had wet it, so that it shone, and made an agreeable Appearance. It diminish'd in Apparent Magnitude as it rose, till it enter'd the Clouds, when it seem'd to me scarce bigger than an Orange, and soon after became invisible, the Clouds concealing it.[2]

The two types of balloon were quickly named after their respective inventors and were designated *Montgolfières* or *Charlières*. They were also known as "fire-balloons" or "air-balloons" according as they were inflated with heated air (fire) or "inflammable air" (hydrogen).

After the ascension of the Charles balloon, in Franklin's words,

> The Multitude separated, all well satisfied & much delighted with the Success of the Experiment, and amusing one another with Discourses of the various Uses it may possibly be apply'd to, among which were many very extravagant. But possibly it may pave the Way to some Discoveries in Natural Philosophy of which at present we have no conception.[3]

It was typical of Franklin to have sufficient foresight to believe that the new invention might serve some useful function. As yet, no passengers had made an ascent and one wonders what use may have occurred to him for a huge bag filled with gas which was let up into the air and soon came down again. Perhaps, with his great interest in meteorological problems, he thought of some method of measuring temperature or wind currents in the upper atmosphere. In any case, we may note that the "use" was not in transportation or communication, but as an instrument paving "the Way to some Discoveries" in *science*.

But, if Franklin thought the new invention might prove useful, there were many who queried what possible use it might serve. In reply, Franklin uttered the now-classic

[2] *Ibid.*
[3] *Ibid.*

remark: What good is a new-born baby? Mr. Carl Van Doren, in his recent splendid biography of Franklin, has shown that there exists contemporary documentary evidence that Franklin actually made this statement, since Baron Grimm hastened to send it to his various royal correspondents in Germany, Poland, and Russia: *Eh, à quoi bon l'enfant qui vient de naître?* [4] As Mr. Van Doren points out, the image was one that rose quite naturally to his mind since there was a baby in his own house at Passy at the very time—little two-weeks old Ann Jay, the daughter of John Jay.

Further evidence that Franklin actually uttered this now-classic expression is afforded by an unpublished letter in the Library of the University of Pennsylvania from Ducarne de Blaugy to Franklin (dated: Hirson, October 3, 1783), according to which the author is "incensed at Dr. Franklin daring to call his balloon an infant just coming to birth." Quoted from "Calendar of the Papers of Benjamin Franklin in the Library of the University of Pennsylvania," (Appendix to vol. IV of "Calendar of the Papers of Benjamin Franklin in the Library of the American Philosophical Society," edited by I. Minis Hays. Philadelphia, 1908) p. 468.

Ever since that time, scientists have made use of Franklin's witty epigram to defend what seems a useless discovery which, in reality, is useless only because it is still newly-born. The most skillful use of Franklin's epigram was made by Michael Faraday in a lecture before the City Philosophical Society in 1816. Speaking on the Subject, "On Oxygen, Chlorine, Iodine and Fluorine," Faraday remarked:

> Before leaving this substance, chlorine, I will point out its history, as an answer to those who are in the habit of saying to every new fact, "What is its use?" Dr. Franklin says to such, "What is the use of an infant?" The answer of the experimentalist would be, "Endeavour to make it useful." When Scheele discovered this substance it appeared to have no use, it was in its infantine and useless state; but having grown up to maturity, wit-

[4] Carl Van Doren: "Benjamin Franklin" (New York, The Viking Press, 1938), p. 700. Chapter 24 of this book contains other interesting material concerning Franklin and the early balloon ascensions.

> ness its powers, and see what endeavours to make it useful have done.[5]

But let us return to the main story concerning balloons.

At Versailles, on September 19, 1783, Joseph Montgolfier sent up a fire-balloon for the first time with a living cargo—a sheep, a cock, and a duck. These first living aerial travellers landed without mishap except for the cock, which sustained an injury of the right wing from a kick delivered by the sheep. About one month later, Pilâtre de Rozier made several ascents in a fire-balloon which was firmly secured to the ground by means of a stout rope, i.e. in a "captive balloon." He demonstrated that one could carry small bundles of straw to be used to replenish the fire burning in a small brazier suspended under the mouth of the bag. Thus it was possible to keep the air within the bag continually warmed and thereby prevent its immediate contraction on the cooling that would otherwise take place immediately upon ascent.

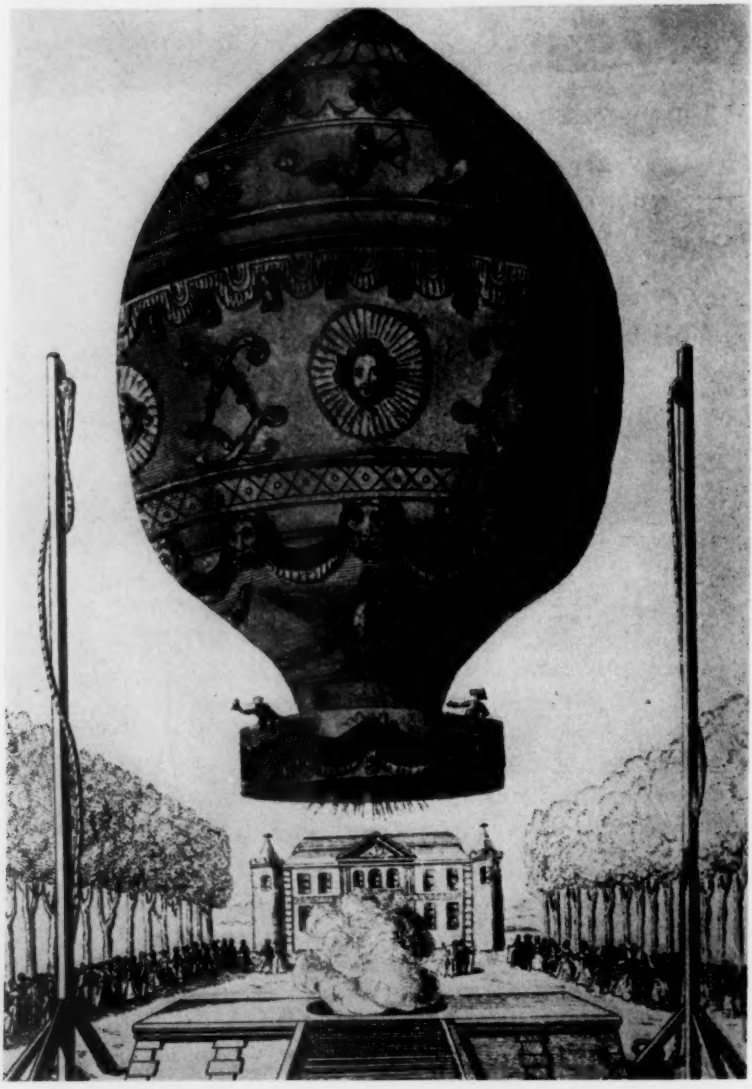
Since the experiments with a captive balloon had proved so successful, de Rozier decided to make a trial in a free balloon. Accompanied by a companion, the Marquis d'Arlandes, he went aloft in a fire-balloon on Nov. 21, 1783, ascending from the Jardin du Château de la Muette. (See Fig. 1.) The balloon rose to a height of about five hundred feet and, after remaining aloft for slightly more than twenty minutes, landed about five miles from the starting point. The balloon was quite a showy affair, covered with extravagant rococo ornaments in brilliant oil colors.

On the same day that he witnessed the ascent of de Rozier, Franklin wrote to Banks:

> I am sorry that this experiment is totally neglected in England, where mechanic genius is so strong. . . . Your philosophy seems too bashful. In this country we are not so much afraid of being laughed at. If we do a foolish thing, we are the first to laugh at it ourselves, and are almost as much pleased with a bonmot or a good chanson, that ridicules well the disappointment of a project, as we might have been with its success. It does

[5] Quoted in Bence Jones: "Life and Letters of Faraday" (London, 1870), vol. I, p. 218.

FIG. 1.



The first ascent of human beings in a free balloon. Pilâtre de Rozier and the Marquis d'Arlandes going aloft in a fire-balloon—*Montgolfière*. Note the extravagant design on the balloon surface, the fire on the platform by means of which the air was heated, and the platform around the mouth of the bag which affords a place for the aeronauts to stand. (From a contemporary print.)

> not seem to me a good reason to decline prosecuting a new experiment which apparently increases the power of man over matter, till we can see to what use that power may be applied. When we have learnt to manage it, we may hope some time or other to find use for it, as men have done for magnetism and electricity, for which the first experiments were mere matters of amusement.
>
> This experiment is by no means a trifling one. It may be attended with important consequences that no one can foresee. We should not suffer pride to prevent our progress in science.[6]

Actual and practical uses suggested themselves immediately to Franklin's fertile mind, once man had made an aerial voyage. To Banks he wrote that these included "elevating an engineer to take a view of an enemy's army, works, etc., conveying intelligence into or out of a besieged town, giving signals to distant places, or the like." [7]

Charles was anxious to show that his type of balloon was easily the equal, if not the superior of the *Montgolfière*. Thus he proceeded to duplicate the feat of Pilâtre de Rozier, and in ten days he was ready. On Dec. 1, 1783, he ascended from Paris in an "air-balloon" accompanied by one of the Robert brothers, who had aided him in its construction. The bag containing the hydrogen was about twenty feet in diameter and suspended beneath it was a car, or "gondola," in which the aeronauts rode. This car was hung from a hoop of iron surrounding the center of the bag and the hoop was, in turn, fastened to a net which covered the upper hemisphere of the balloon proper. Charles and Robert ascended from the Tuilleries garden at a quarter to two in

[6] Franklin to Sir Joseph Banks, Passy, Nov. 21, 1783. "Writings of B. F." (*ed. cit.*), vol. ix, pp. 113–118. An example of the ridicule of ballooning by the English may be found at the end of this paper. The first aerostatic experiments in England were made by Count Francesco Zambeccari. George III, who was interested in the experiments, wrote in October of 1783 to Sir Joseph Banks, offering to defray the cost of balloon experiments if the Royal Society thought thay any useful purpose could be served thereby. Franklin's letters to Banks had been without effect, as it would seem, because the Council of the Society agreed that "no good whatever could result." Cf. J. E. Hodgson: "The History of Aeronautics in Great Britain," (London, Oxford University Press, 1924), p. 113.

[7] *Ibid.*

the afternoon and, after remaining for some two hours at an elevation of about two thousand feet, descended to the ground at the small town of Nesle, some twenty-seven miles from Paris. Here Robert alighted and Charles proceeded to make a second ascent. He remained aloft by himself for about a half hour and then descended a few miles outside the town. (See Fig. 2.) On the second ascent, he neglected

FIG. 2.

Charles' second landing at Nesles on December 1, 1783. Note the method of suspending the car from the bag, the valve at the bottom of the bag proper by which ascent and descent were controlled. Note too the comparative ease and grace with which Charles is making his landing. (From a contemporary print.)

to add ballast to compensate for the weight of his former co-traveller, with the result that his ascent was extremely rapid and caused him violent pain in his right ear and in his jaw.

All of the modern features of the hydrogen balloon are due to Charles' invention. In addition to the means of suspending the car from the bag and the use of hydrogen rather than heated air, Charles invented the valve at the mouth of the bag which gave the aeronaut control of the ship during ascent and descent.

Franklin, writing to Banks, compared the two types of balloon as follows:

> This method of filling the balloon with hot air is cheap and expeditious. . . .
>
> The other method of filling a balloon with permanently elastic inflammable air, and then closing it, is a tedious operation, and very expensive. . . . [Aeronauts riding in the car suspended from this kind of balloon] will have more leisure . . . than the others, having no fire to take care of. They say they have a contrivance which will enable them to descend at pleasure. I know not what it is, but the expense of this machine, filling included, will exceed, it is said, ten thousand livres.
>
> This balloon of only twenty-six feet diameter, being filled with air ten times lighter than common air, will carry up a greater weight than the other, which though vastly bigger, was filled with an air that could scarcely be more than twice as light. Thus the great bulk of one of these machines, with the short duration of its power, and the great expense of filling the other, will prevent the invention being of so much use as some may expect, till chemistry can invent a cheaper light air producible with more expedition.
>
> By the emulation between the two parties running high, the improvement in the construction and management of the balloons has already made a rapid progress, and one cannot say how far it may go. A few months since the idea of witches riding thro' the air upon a broomstick, and that of the philosophers upon a bag of smoke, would have appeared equally impossible and ridiculous.[8]

Several months later, Franklin wrote of the same problem to Ingenhousz and ended by noting that the expense of producing the hydrogen was being diminished by the work of the chemist, Guyton de Morveau. The Montgolfier type of balloon, Franklin wrote, "is sooner and cheaper filled; but must be bigger to carry up the same weight; since Air rarified by Heat is only twice as light as common Air, and inflammable Air

[8] *Ibid.*

is ten times lighter. M. de Morveau, a famous chemist at Dijon, has found an inflammable Air that will cost only a 25th part of the Price of what is made by oil of Vitriol poured on Iron Filings. They say it is made from Sea Coal." [9] As we shall see presently, Morveau's invention was to make military ballooning both practical and feasible.

After witnessing the ascent of Charles and Robert, Franklin went home and immediately wrote an account of the event to Banks. As in the previous instance, Franklin communicated the news to the President of the Royal Society on the very day on which the event took place. He was not only a reliable observer, but a punctual one in the bargain. He informed his English correspondent that "the travellers had perfect command of their carriage, descending as they pleased by letting some of their inflammable air escape, and rising again by discharging some sand." [10]

More ascents were made and on Dec. 16, 1783, Franklin wrote to Henry Laurens, "We think of nothing here at present but of Flying; the balloons engross all attention." Previsaging the immediate possibility of crossing the English Channel by air, he added that "Messrs. Charles and Robert made a trip last Monday thro' the Air to a place farther distant than Dover is from Calais." The progress in ballooning was very rapid, he noted, "yet I fear it will hardly become a common Carriage in my time, tho' being the easiest of all Voitures it would be extreamly convenient to me, now that my Malady forbids the Use of the old ones over a Pavement." [11] Some years later, when he had returned to Philadelphia, when walking was still a painful exercise for him, and riding in a carriage over the rough streets of Philadelphia was more painful still, this idea returned to him and he wrote, half wistfully and half humourously, to his friend Jean Baptiste Leroy in Paris: "I had sometimes wished I had brought with me from France a balloon sufficiently large to raise me from the ground.

[9] Franklin to Jan Ingenhousz, Passy, Jan. 16, 1784. "Writings of B. F." (*ed. cit.*), vol. ix, pp. 155–156.

[10] Franklin to Sir Joseph Banks, Passy, Dec. 1, 1783. "Writings of B. F." (*ed. cit.*), vol. ix, pp. 119–122.

[11] Franklin to Henry Laurens, Passy, Dec. 6, 1783. "Writings of B. F." (*ed. cit.*), vol. ix, pp. 122–123.

In my malady, it would have been the most easy carriage for me, being led by a string held by a man walking on the ground." [12]

Philadelphia had had its own aerostatic experiments and adventures. After the news of the successful experiments of the Montgolfiers and Charles had reached the city from Franklin in Paris, David Rittenhouse and Francis Hopkinson, both members of Franklin's American Philosophical Society, began experiments of their own. They varied the form of Charles' machine and constructed a vehicle consisting of a car attached to forty-seven small bags which were filled with hydrogen. After several experiments of ascents in which the machine was held captive, a local carpenter, James Wilcox, was induced to make a free flight. This flight lasted only for ten minutes when, through fear of falling into the Schuylkill River, Wilcox effected a speedy descent by puncturing several of the forty-seven small balloons.

Franklin's prevision of an air crossing of the Channel became reality on January 7, 1785, when Dr. John Jeffries and Pierre Blanchard ascended from Dover in a hydrogen balloon of twenty-seven feet diameter and landed successfully in the forest of Guinnes in France. Although Jeffries was an American "loyalist" (Surgeon-General in the British army) during the Revolution, Franklin entertained him several times in his house at Passy. The first of these visits was made exceedingly pleasant for Franklin by the fact that Jeffries brought him a letter from London, described by Franklin as "the first thro' the Air." [13] The first "air-mail" letter was sent to Benjamin Franklin.

In considering the uses and effects of the new invention, Franklin saw clearly that it would very likely "give a new turn to human Affairs." Writing to Ingenhousz, he declared:

> Five thousand Balloons, capable of raising two Men each, could not cost more than Five ships of the Line; and where

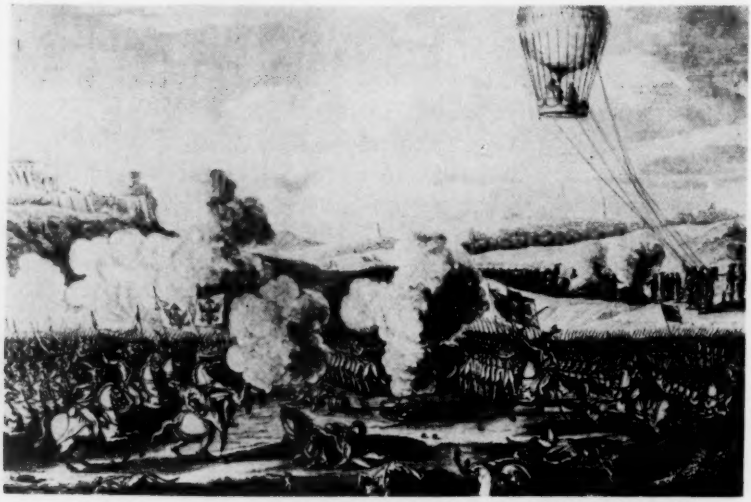
[12] Franklin to Jean-Baptiste Le Roy, Philadelphia, April 18, 1787. "Writings of B. F." (*ed. cit.*), vol. ix, pp. 572–573.

[13] Franklin to James Bowdoin, Philadelphia, Jan. 1, 1786. "Writings of B. F." (*ed. cit.*), vol. ix, pp. 479–481. Cf. Carl Van Doren: "Benjamin Franklin" (*op. cit.*), p. 702; also cf. John Jeffries' Diary, edited by B. Joy Jeffries in *The Magazine of American History*, XIII (1885), pp. 66–88.

> is the Prince who can afford so to cover his Country with Troops for its Defense, as that Ten Thousand Men descending from the Clouds might not in many places do an infinite deal of mischief, before a Force could be brought together to repel them? [14]

The Revolution was over and won, and Franklin was quite naturally anxious to find some way of preserving world peace. He thought that one effect of the ballon might be to convince

FIG. 3.



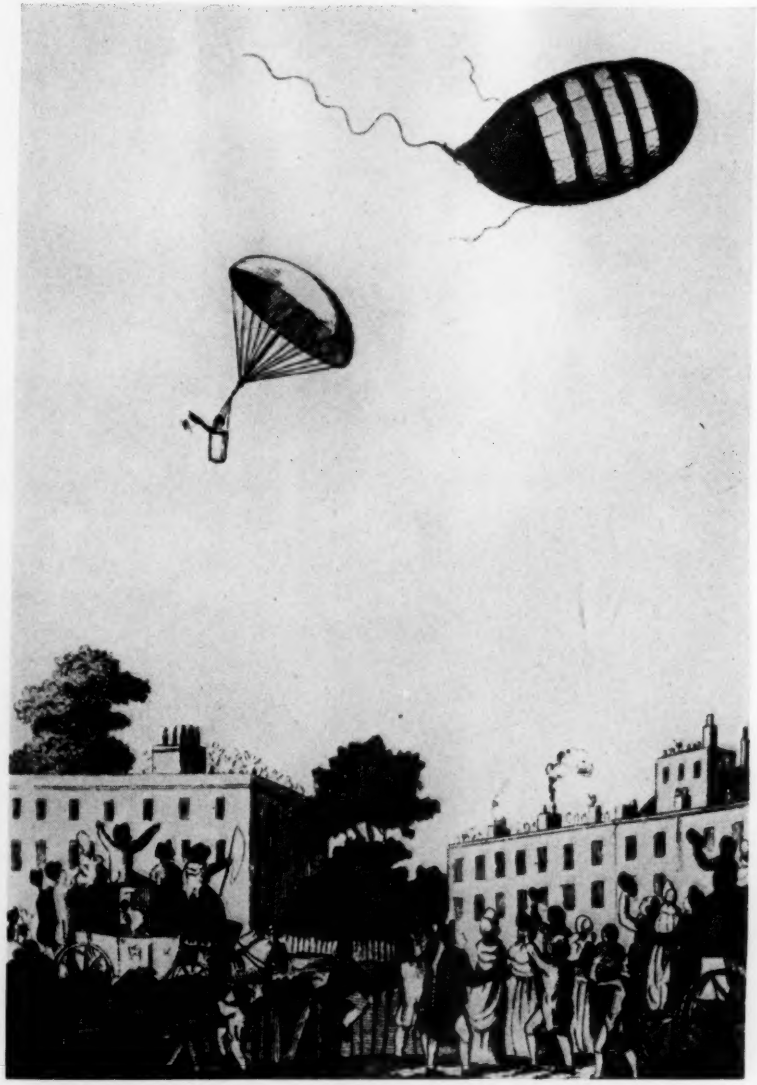
The first use of an air-ship during a battle. Colonel Coutelle and General Morlot observing the progress of the Battle of Fleurus in June 1794. (From a contemporary print.)

"Sovereigns of the Folly of wars," since, owing to the possibility of an air invasion as envisaged in the quotation above, "it will be impracticable for the most potent of them to guard his dominions."

Military aviation began soon after the French Revolution, with the founding of a school of army aeronautics at Meudon under the direction of the distinguished chemist, Guyton de Morveau, and Colonel J.-M.-J. Coutelle of the French army.

[14] Franklin to Jan Ingenhousz, Passy, Jan. 16, 1784. "Writings of B. F." (*ed. cit.*), vol. ix, pp. 155–156.

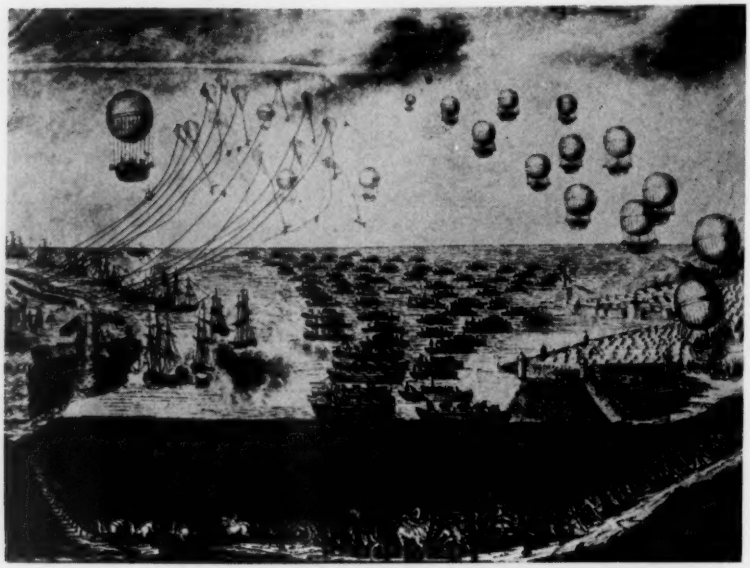
FIG. 4.



The first parachute landing was made in England by a Frenchman. "Mons. Garnerin" descending from a height of about 800 feet. (From a contemporary print.)

Four captive air-balloons were constructed and staffed, and then assigned to the four armies in the field: the army of the north, the army of the Sambre and the Meuse, the army of the Rhine and the Moselle, and the army of Egypt. Just before the battle of Fleurus in Flanders (June 1794), Col. Coutelle and the commanding officer of the French army made two reconnaissance ascents of lasting four hours each in order

FIG. 5.

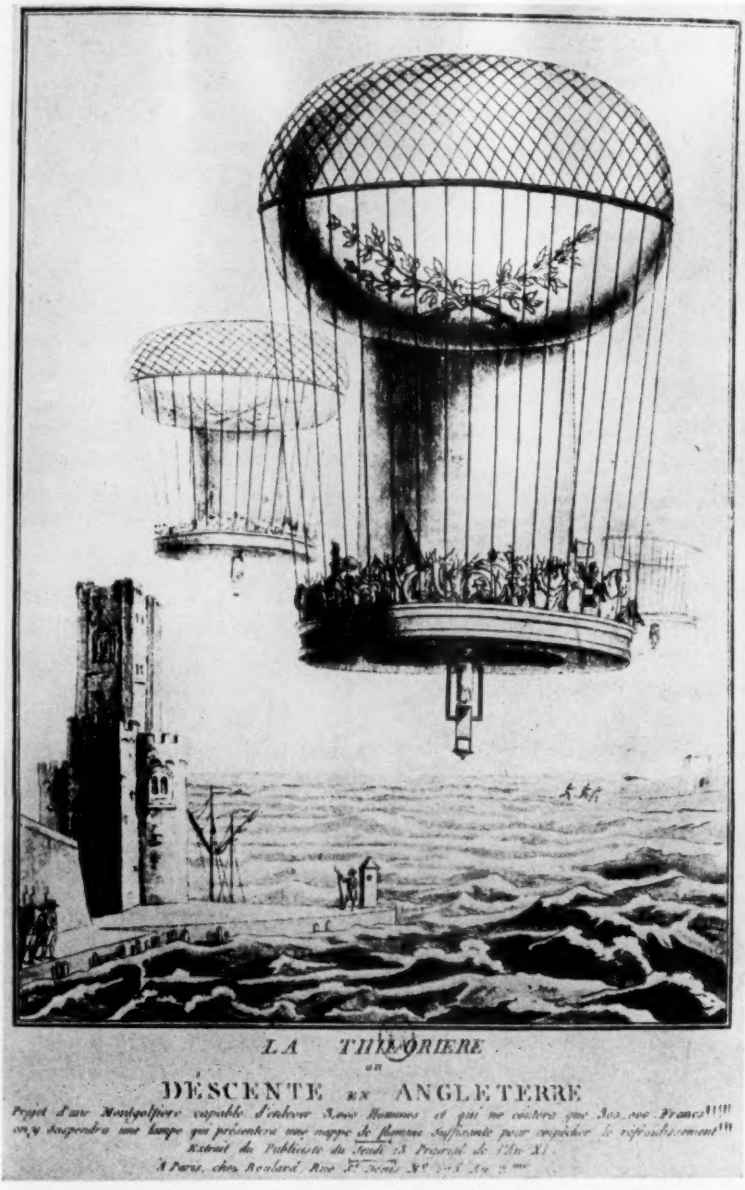


Three-way invasion of England, 1804 style. The French are descending upon England from Boulogne-sur-Mer. The English fleet is firing at the oncoming barges. British sharpshooters, attached to kites, are shooting at the oncoming air-balloons, but the marching infantry and horse-drawn artillery move unnoticed and unhampered through the tunnel. (From a contemporary print.)

to study and to reconnoitre the opposing army. (See Fig. 3.) The French victory on the following day was due, in large measure, to the complete knowledge of the disposition of enemy troops and artillery which had been gained by the French high command from its aerial observation. The balloon corps was, thereafter, in continual demand throughout the rest of the campaign.

But if Franklin's first idea concerning the use of balloons for purposes of military observation became reality in short

FIG. 6.


LA THILORIERE
ou
DESCENTE EN ANGLETERRE
Projet d'une Montgolfière capable d'enlever 3,000 Hommes et qui ne coûtera que 300,000 Francs!!!!!
on y suspendra une lampe qui présentera une nappe de flamme suffisante pour empêcher le refroidissement!!!
Extrait du Publiciste du Jeudi 13 Prairial de l'An XI.
A Paris, chez Roulard, Rue St. Jacq. N°. 54 Au 2me

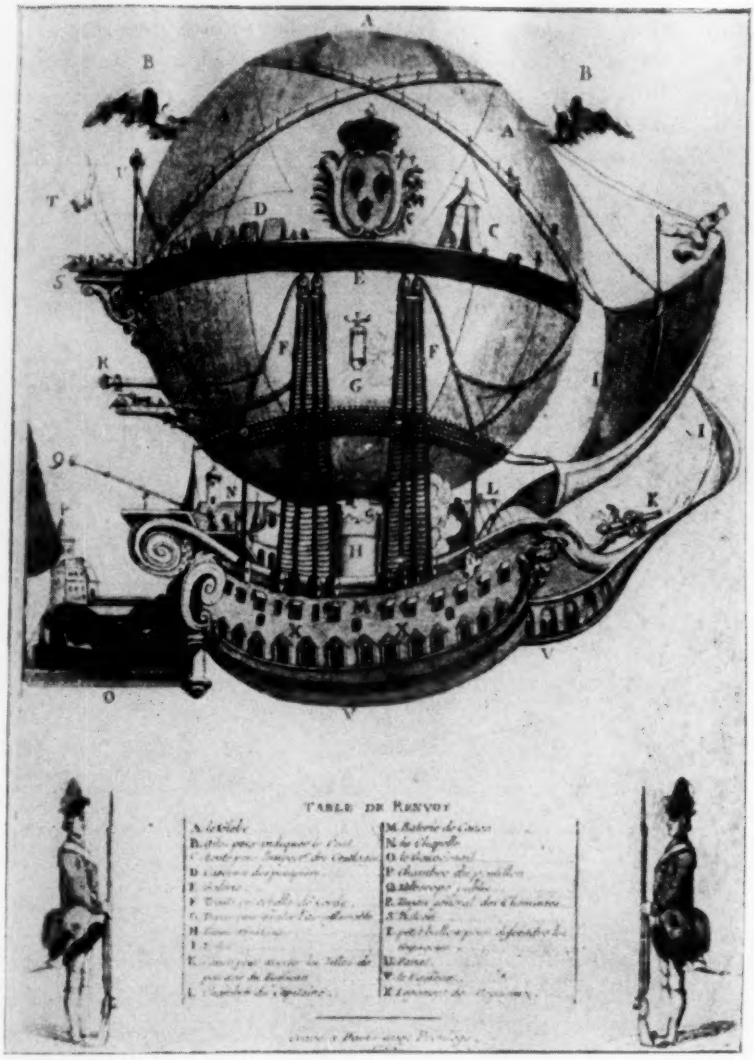
This method of air-invasion of England was put forward by a French lawyer, Thilorier, in the *Publiciste* (Paris) of June 2, 1803. The plan was to construct a series of *Montgolfières*, each capable of carrying 3,000 men and equipment and each costing about 300,000 francs. Named *Thiloriers* in honor of their inventor, these balloons were never constructed, although plans for subscription were drawn up. Note the lamp suspended beneath the platform to protect the men from excessive cold. Three Englishmen on the opposite shore of the Channel are fleeing in terror. (From a contemporary print.)

order, his idea of an air invasion remained curiously visionary. Probably the great difficulty on controlling the landing of a free air-balloon was an insurmountable obstacle, and the use of parachute troops awaited the Second World War which today occupies most of the civilized countries of the earth. Parachutes themselves, however, were introduced at the very start of the nineteenth century when, in 1802, a Frenchman—André-Jacques Garnerin—made a successful parachute landing in London. (See Fig. 4.)

During the period of the Napoleonic wars, many French plans were conceived which included an invasion of England from the air. One plan involved a three-way invasion: (1) by use of barges to transport troops and equipment across the Channel, (2) by means of a tunnel under the Channel, and (3) by the use of *Charlières*—hydrogen balloons—of an enormous size to allow for the transportation of troops. (See Fig. 5.) Another plan involved the construction of large *Montgolfières*—fire-balloons—to enable troops to fly over the Channel and successfully invade the British Isles without the interference of the British Navy. (See Fig. 6.) A move was actually begun to gather subscriptions to put this last project into practical application.

Despite many artists' conceptions of the use of balloons as actual ships of war (see Figs. 7 and 8), nothing further was done in this line until 1849, when the imperial Austrian army was laying siege to the free Republic of Venice. Then, an enterprising Austrian engineer, who had been detailed to the company of observers who made daily ascents in captive balloons to see the state of the city's defenses, decided to bomb Venice from the air. He released small free-balloons loaded with bombs which were controlled by time-fuses. The first attempt was frustrated by adverse winds which arose soon after the balloons were released, with the effect of blowing the balloons and their deadly cargo out over the sea where they caused no damage. A second attempt resulted in a violent explosion of the bombs in the gardens outside the Doge's Palace. No one was injured and little damage was caused; the only advantage gained thereby by the Austrians was a possible disruption of republican morale.

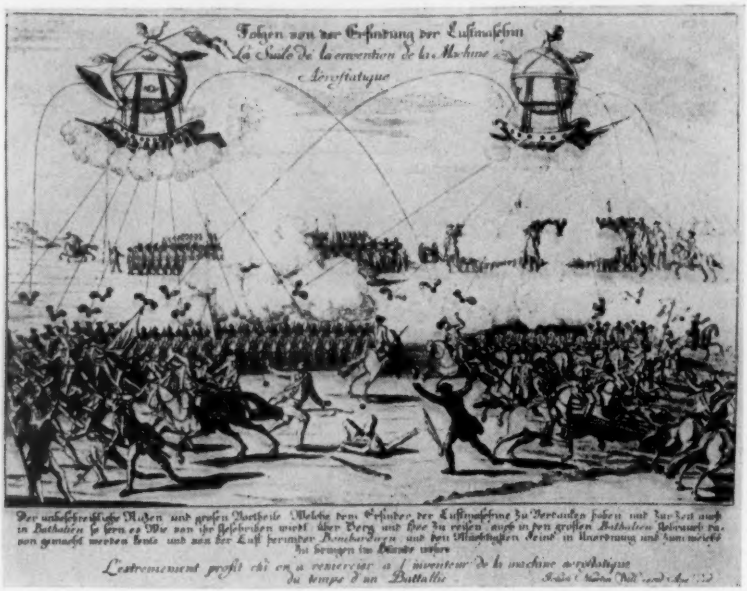
FIG. 7.



Artist's conception of a royal air-ship, 1774 style. This fantasy was redrawn many times during the last fifteen years of the eighteenth century but was never constructed. Note sails, the gun mounted in the bow of the suspended ship, telescope in rear of ship, large buildings on ship's deck. The little building on the rear deck (*N*) is a chapel. On the ledge running around the "equator" of the bag there are small tents for soldiers and a large tent for the officers. (From a contemporary print.)

In the Franco-Prussian War, balloons played an important part. During the siege of Paris (1870–1871) the only communication between the besieged city and the rest of the country was provided by balloons. Sixty-four balloons were released between Sept. 23, 1870 and Jan. 28, 1871. Each balloon carried men and despatches as well as pigeons to bring

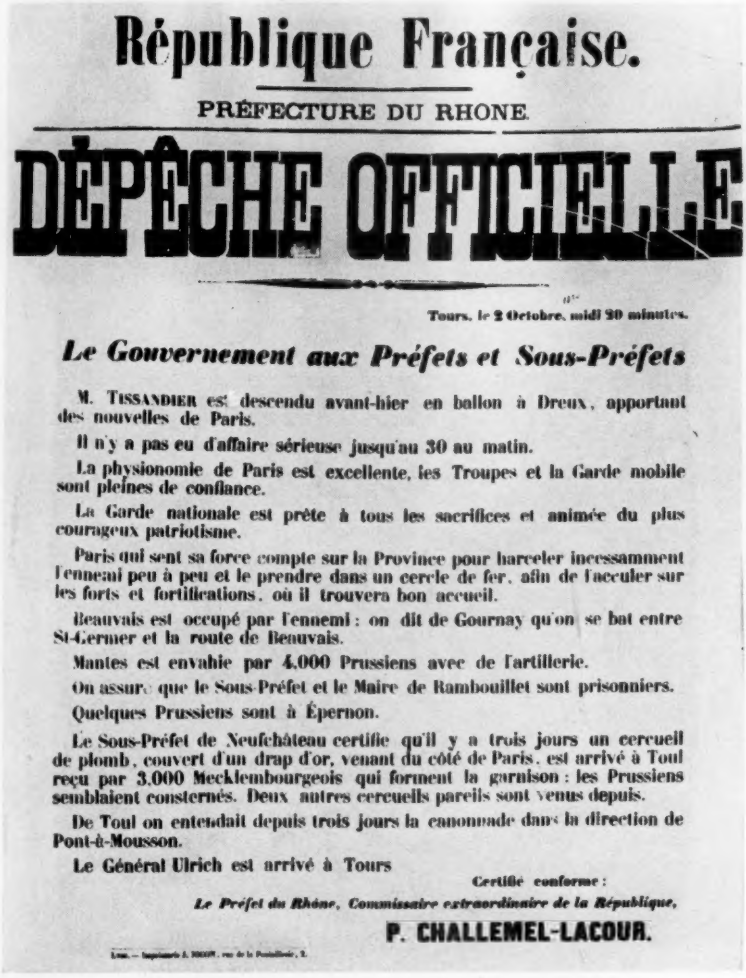
FIG. 8.



German cartoon issued soon after the Battle of Fleurus caricaturing the French military use of balloons and affording a satirical prevision of the future use of air-ships to bombard land forces. Note that each balloon has, suspended under the bag, a regular ship, and that the cannon protrude from port-holes or gun-ports in the sides. (From a contemporary print.)

return messages back to the city. The service was organized under the direction of the Post Office and the aeronauts were usually sailors in the French navy. (See Figs. 9 and 10.) Of the sixty-four balloons released, only two were never heard from again; presumably they drifted out over the ocean and were lost. Several balloons, however, were shot down by the Prussians. And, finally, Leon Gambetta made his escape from the city in a balloon on Oct. 7, 1781, thereby undoubtedly prolonging the war. Thus Franklin's clear view that balloons

FIG. 9.



République Française.

PRÉFECTURE DU RHONE.

DÉPÊCHE OFFICIELLE

Tours, le 2 Octobre, midi 30 minutes.

Le Gouvernement aux Préfets et Sous-Préfets

M. TISSANDIER est descendu avant-hier en ballon à Dreux, apportant des nouvelles de Paris.

Il n'y a pas eu d'affaire sérieuse jusqu'au 30 au matin.

La physionomie de Paris est excellente, les Troupes et la Garde mobile sont pleines de confiance.

La Garde nationale est prête à tous les sacrifices et animée du plus courageux patriotisme.

Paris qui sent sa force compte sur la Province pour harceler incessamment l'ennemi peu à peu et le prendre dans un cercle de fer, afin de l'acculer sur les forts et fortifications, où il trouvera bon accueil.

Beauvais est occupé par l'ennemi : on dit de Gournay qu'on se bat entre St-Germer et la route de Beauvais.

Mantes est envahie par 4,000 Prussiens avec de l'artillerie.

On assure que le Sous-Préfet et le Maire de Rambouillet sont prisonniers.

Quelques Prussiens sont à Épernon.

Le Sous-Préfet de Neufchâteau certifie qu'il y a trois jours un cercueil de plomb, couvert d'un drap d'or, venant du côté de Paris, est arrivé à Toul reçu par 3,000 Mecklembourgeois qui forment la garnison : les Prussiens semblaient consternés. Deux autres cercueils pareils sont venus depuis.

De Toul on entendait depuis trois jours la canonnade dans la direction de Pont-à-Mousson.

Le Général Ulrich est arrivé à Tours

Certifié conforme :

Le Préfet du Rhône, Commissaire extraordinaire de la République,

P. CHALLEMEL-LACOUR.

Lyon. — Imprimerie A. REGEY, rue de la Poulaillerie, 2.

Official poster, published at Lyons, announcing the safe arrival of a balloonist from Paris with the latest news from the city, some of it good, some bad.

would serve an invaluable function in communication when a city was besieged became reality after about a hundred years.

In the American Civil War, balloons were used quite extensively by the Union Army under McClellan's command.

FIG. 10.

Prix maximum du Journal : 3 gr. 20 c. Le Numéro, 20 centimes. N° 8. — Dimanche 27 Novembre 1870.

LE BALLON POSTE

Journal du Siége de Paris, publié pour les Départements et l'Étranger.

Ce Journal, de petit régénérateur, est destiné à être expédié en Province pour 20 centimes. Il paraît deux fois par semaine, le Dimanche et le Jeudi, avec les nouvelles des sept jours précédents.

Ce n'est donc qu'au bout de ce temps que sa rédaction se trouve entièrement changée, et un autre hebdomadaire suffit pour tenir les Départements au courant de tout ce qui se passe à Paris.

DIRECTEUR GÉRANT : Gabriel Richard

Aux bureaux de la Publication, 19, rue des Martyrs, Paris.

Le Journal renferme deux colonnes blanches, destinées à la correspondance privée.

On peut se procurer, aux bureaux de la Publication, les premiers numéros du Journal, et des ouvrages généraux destinés à s'orienter et à faciliter vos correspondances, sans dépasser le poids réglementaire.

La publication du Journal le BALLON-POSTE n'est pas limitée à la durée du siége de Paris.

Ce Journal continuera à paraître, après la levée du siége, dans les mêmes conditions typographiques, de façon à pouvoir se placer dans les lettres expédiées dans Paris pendant et à l'époque, en se surchargeant que fortement leur poids. Son titre, son prix et son mode de publication subiront alors quelques modifications, rendre dans l'intérêt de lecteurs. On peut faire partir de n'importe quelle date l'abonnement à la série de dix numéros.

LE BALLON-POSTE

Journal du siége de Paris

Du Dimanche 20 au Samedi 26 novembre 1870.

Dimanche 20 novembre. — Le gouvernement met à l'ordre du jour les noms des défenseurs de Paris, appartenant à la garde nationale, à l'armée de terre et de mer, à la garde mobile et aux corps francs, qui ont bien mérité du pays depuis le commencement du siége. Ce témoignage de la gratitude publique nous inscrit par leurs états de service. Saluons les noms de nos citoyens.

— ARRÊTÉ : Le général commandant supérieur du génie de l'armée de Paris et le directeur des fortifications sont autorisés à exercer le droit de réquisition pour tout ce qui concerne le logement, des troupes et le matériel de guerre, les occupations du terrain et les approvisionnements de matériaux et d'objets nécessaires aux travaux de défense.

— Régiment choisissant, dans chaque arrêté, un hôpital qui prendra le titre de *Régimentaire*, pour distribuer les blessés en cas d'action entre les diverses ambulances du même quartier. Les hôpitaux choisis sont les suivants : 1er secteur : Hôpital Saint-Antoine. — 2e secteur : Hôpital Saint-Louis. — 3e secteur : Hôpital Saint-Martin. — 4e secteur : Hôpital Lariboisière. — 5e secteur : Hôpital Beaujon. — 6e secteur : Hôpital du Gros-Caillou. — 7e secteur : Hôpital Necker. — 8e secteur : Hôpital du Val-de-Grâce. — 9e secteur : Hôpital de la Pitié.

Lundi 21 novembre. — Arrêté du gouverneur de Paris interdisant tout affichage et placards de journaux, bouilles publiques ou autres politiques de même nature.

Cet arrêté se base sur la violation des lois sur l'affichage et sur le droit donné par l'état de siége aux chefs militaires d'interdire les publications de nature à exercer sur l'armée et les citoyens une influence pernicieuse.

Il est certain que depuis quelque temps la publicité de l'affiche était entrée dans une nouvelle voie. Quelques journaux faisaient afficher leurs exemplaires du jour ou de la veille, afin de se faire connaître ou de propager leurs opinions dans le monde de ceux qui n'avaient pas les moyens de les acheter.

C'est à cette propagande qu'on a voulu mettre un terme, bien qu'elle fût exercée par des journaux de toutes nuances. Cet acte de répression est diversement jugé : mais on le regarde comme sévère. — Il ne s'accorde pas avec la liberté de publication que la République semble nous promettre. Il faut laisser au bon sens public le soin de faire justice des plus ardu simples et mal intentionnés. Le Paris du 3 novembre tuerait plus de confiance.

— Décret ordonnant la formation de deux bataillons de chasseurs à pied, de sept compagnies chacun, portant les numéros 21 et 22.

Le même décret supprime le 4e bataillon du 19e régiment d'infanterie de ligne, qui passe dans les bataillons de chasseurs ci-dessus.

Création d'une 7e compagnie dans chacun des trois bataillons du 4e régiment de marche.

— Le Gouvernement reçoit un nouveau journal paraissant qui s'imprime à Versailles, sous le nom de *Moniteur de département de Seine-et-Oise*, et le fait insérer in extenso dans le *Journal officiel*. Paris n'est pas sous de ce nouveau hyperbole, aveu dans la note heureuse de nos ennemis. La meilleure manière d'y répondre et d'en faire remettre les absurdités était certainement de le publier. On voit que ses auteurs, en exécutant de vouloir la République, sont loin d'être convaincus de ce qu'ils avancent de persuader aux autres.

On trouve, dans ce journal, la circulaire de M. de Bismark, du 6 novembre, rendant compte de son entrevue avec M. Thiers, quelques extraits tronqués de journaux français et anglais, et des faits divers à l'honneur de la Prusse.

— Les rapports militaires ne contiennent que quelques combats heureux d'avant-postes, à Villeneuve, et une surprise opérée par des Prussiens déguisés, qui ont fait une centaine d'arrestations du 1er régiment d'éclaireurs.

Mardi 22 novembre. — Décret portant réquisition de toutes les pommes de terre, existant dans Paris et dans la banlieue, en dehors des provisions de ménage. — Les détenteurs sont tenus, sous peine de confiscation, de faire leur déclaration dans les cinq jours.

— Création de quatre nouvelles batteries à pied dans le régiment d'artillerie de marine.

— Arrêté supprimant, à partir du 30 novembre, les livraisons de gaz aux particuliers et aux établissements publics.

Cette dernière mesure était depuis longtemps prévue, et l'on s'étonne qu'elle n'ait pas été prise plus tôt, car il existe à Paris un approvisionnement considérable d'huiles minérales qui peut parfaitement suppléer au gaz d'éclairage.

— *Le Journal officiel* publie une circulaire à tous les agents de la France à l'étranger, en réponse à celle de M. de Bismark que nous a apportée le *Journal de Seine-et-Oise*.

Le ministre des affaires étrangères y rétablit les faits que M. de Bismark *se plaît à dénaturer* « par une présomption très contraire d'ailleurs à toute *sa politique antérieure*. » Il revient sur la question du ravitaillement qui avait été accepté par la Prusse au début des négociations et « qu'elle n'aurait pas tenté d'ajourner sous la validation de Metz et la fameuse journée du 27 octobre. » Il établit l'intérêt et le droit qu'a tout jours eu le Gouvernement de voir « l'opinion du Peuple française s'exprimer librement par l'élection d'une représentation nationale. »

Mais il fait ressortir l'impossibilité d'organiser des élections loyales et respectées sans armistice régulier et sans la pression matérielle.

M. Jules Favre fait justice de la calomnie qui représente le gouvernement « prenant à parler à une république qu'il aurait eu établir l'oubliette de force. Il rappelle hautement une pareille imputation. *Il a accepté avec empressement les mêmes et c'est hautement effacer de la face* [illegible] *A cette heure impulse, et ces remparts volontaires du jugement de ceux dont la voix bienveillante n'a pas été écoutée. Ce n'est pas d'eux que lui viendrait un conseil de désespoir.* »

Cette dernière phrase nous remarquons. Elle n'est point un signe de faiblesse, on annonce où la victoire d'hérissons semble couronner la fortune sous son drapeau.

— Les compagnies de guerre du 2e bataillon de la garde nationale se sont présentées à l'Hôtel-de-Ville et y ont été passées en revue par M. Eugène Pelletan.

Les rapports militaires signalent une vive fusillade, appuyée par le canon des forts, sur le front de nos lignes du sud.

— L'Académie des sciences s'occupe toujours des ballons dirigeables, sans que personne que le succès ait pu encore récompenser. M. Dupuy de Lome travaille activement à l'adresse qu'il prétend conduire ; celui de M. Yon est en construction depuis plusieurs mois. — Le général officier qui cite guerre aux jours derniers. Menacé d'un M. Thibaudier, qui prétend avoir trouvé le secret de l'aviation, et qui offre pour [illegible] de faire traverser [illegible] à la France. Ce n'est pas cher, mais encore faudrait-il que ce fût sérieux.

Il est certain qu'on peut obtenir, dans un air calme, une certaine vitesse de translation, mais qui ne peut lutter contre le vent. Les théâtres de MM. Guillard, Perin et Moliez ont donné à cet égard des renseignements qui prouvent que de vrai consulter les aéronautes.

Mercredi 23 novembre. — Journal ordre du jour signalant les défenseurs de Paris à qui leur belle conduite devant

CORRESPONDANCE PRIVÉE

Siege newspaper, published twice weekly in besieged Paris. The third column was left blank so that those who wished to send copies of this news-sheet to friends or relatives outside Paris could have a place for their letter or personal message. The day after these were printed, they were sent out by balloon couriers organized by the post office.

The balloon corps was under the command of its founder, Prof. Thaddeus S. C. Lowe, an enterprising aeronaut who had begun to make ascensions as early as 1856. Soon after the outbreak of the Civil War, Lowe went to Washington and submitted plans for the organization of a military air corps

to serve as observers. Secretary of War Cameron was interested in the proposal but seems to have doubted its practicability and referred the matter to the distinguished scientist, Joseph Henry, Secretary of the Smithsonian Institution. While awaiting his decision, Lowe made several ascensions in the capitol, during one of which (June 1, 1861) he sent to Abraham Lincoln the first telegraph message ever sent from a balloon. The message read:

Balloon *Enterprise*,
June 1, 1861.

TO THE PRESIDENT OF THE UNITED STATES:

Sir:

This point of observation commands an area nearly 50 miles in diameter. The city, with its girdle of encampments, presents a superb scene. I have pleasure in sending you this first dispatch ever telegraphed from an aerial station, and in acknowledging indebtedness for your encouragement for the opportunity of demonstrating the availability of the science of aeronautics in the military service of the country.

T. S. C. LOWE [15]

The ascension had been made in a captive balloon and the telegraph wires from the balloon to the ground were entwined around the anchor rope. Henry gave Lowe's project his unqualified approval and an aeronautic section of the Army of the Potomac was soon established and rendered continuous valuable service for some two years—from the Battle of Bull Run to the Battle of Gettysburg.

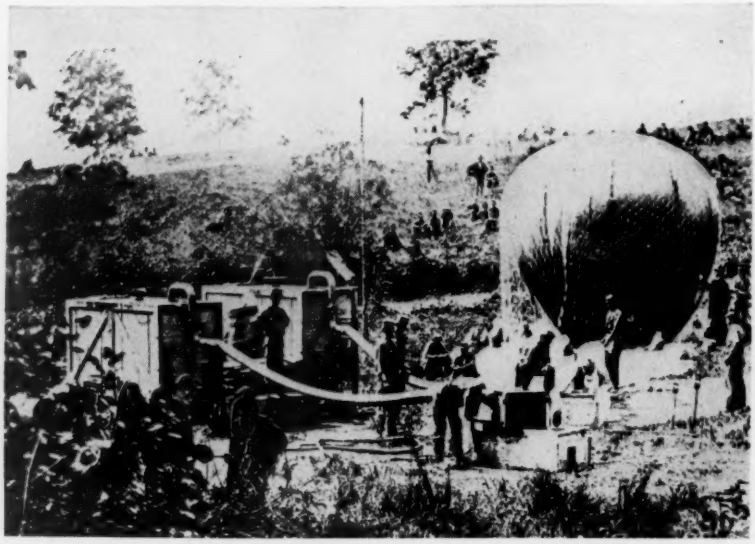
The balloon company was composed of some fifty non-commissioned officers and men, Lowe himself, who was Captain, and a junior officer. Two captive balloons were used, each drawn by a team of four horses. There were two hydrogen generators which were likewise each drawn by four horses and, finally, there was an acid tank-cart drawn by two horses. (See Fig. 10.) When in use, each balloon

[15] Cf. the volume which has just appeared, written by F. Stansbury Haydon: "Aeronautics in the Union and Confederate Armies" (Baltimore, The Johns Hopkins Press, 1941), vol. I, p. 175. This message was published in various newspapers in Washington, Boston, Detroit, New York, Philadelphia, etc.

usually contained from thirteen to twenty-six thousand cubic feet of hydrogen. Inflation of the varnished silk bag required in the neighborhood of three hours if both generators were used simultaneously. The company was photographed in action by Matthew Brady, the distinguished photographer of the Civil War.

Lowe not only made reconnaissance ascents before the battle, but he also remained aloft throughout the progress of

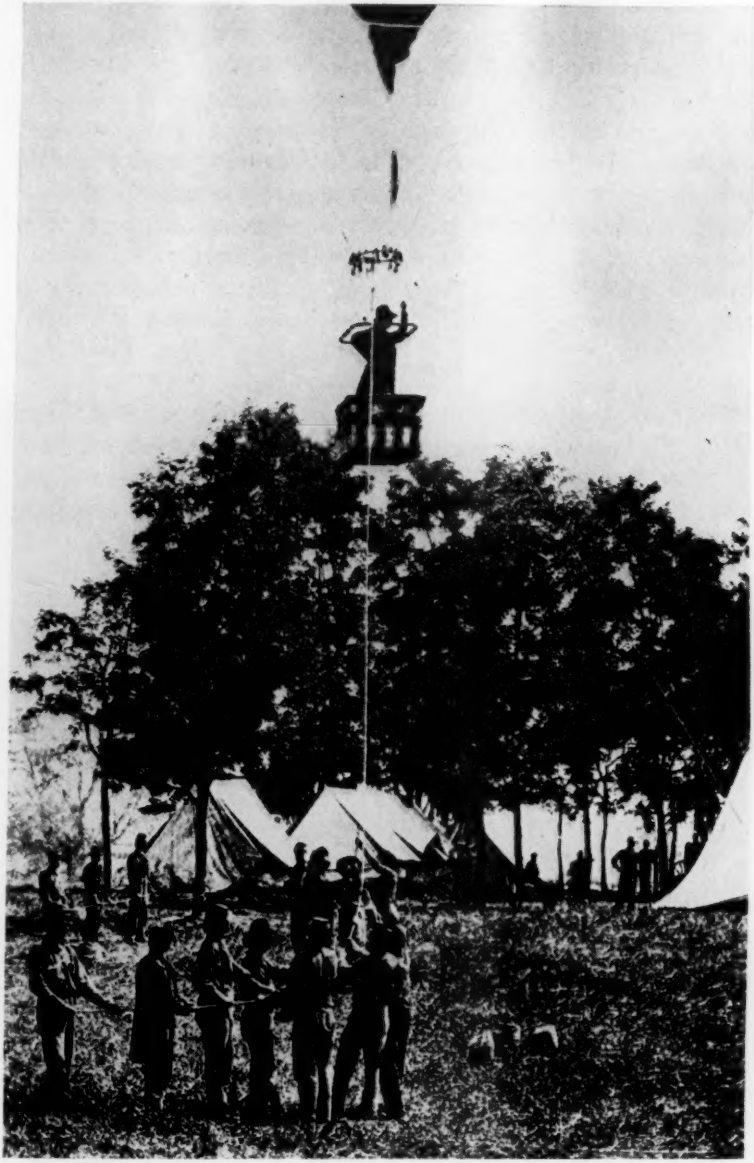
FIG. 11.



Prof. Lowe's crew inflating one of his two observation balloons during the American Civil War. This first American air-corps was part of the Union Army under McClellan's command. (Photo by U. S. Army Signal Corps. Made by Matthew Brady.)

the fighting. By means of a telegraph system, he kept in constant communication with General Head-Quarters which was thus supplied with last minute news of the engagement and was able to make its decisions in accordance with that information. (See Fig. 11.) Lowe was the first person in America to make photographs from the air. After 1863, however, the topography of the theatre of war made balloon observation more difficult, if not impossible, and increased the difficulty of the transportation problem so that the balloon

FIG. 12.



Prof. Lowe observing the Battle of Seven Pines. Note that his observation basket is decorated with the Union ensignia. A cable runs from the basket to the tent, enabling him to keep in constant telegraphic communication with his superiors at General Head-Quarters. During this battle, the telegraph lines from the balloon were connected, via Fortress Monroe, directly to the War Department office in Washington. (Photo by U. S. Army Signal Corps. Made by Matthew Brady.)

company could no longer justify the expense of its operation. Military ballooning became thus impractical and the first American military air corps was soon disbanded.

One last item remains to be introduced, an interesting article which has, thus far, never been incorporated into the history of aviation or the history of America. And, although it concerns Benjamin Franklin, it has never, to my knowledge, been incorporated into the literature concerning him. The item in question is a letter which was published in *The General Evening Post*, a London newspaper, in issue no. 7755, Nov. 11, 1783.[16] The letter is not signed and reads *in extenso* as follows:

Extract of a Letter from Paris, Oct. 16.

> I am truly sorry to inform you, that the new invention of the air balloon has been the cause of a fatal accident, which (though entirely owing to the mal-adresse of the sufferer) may probably damp for a while that ardour of investigation with which so noble a discovery ought to inspire the lovers of philosophical experiments. One Joseph Fathom, an American amanuensis to the celebrated Dr. F——, who has often had hair-breadth escapes from the dangers into which he was betrayed by his eagerness in the imitations of his master's trials, had no rest night or day, till he had constructed at Passy, an air balloon of an immense size, for a purpose about which he observed the strictest secrecy. This machine was fastened to the ground by two strong bars of iron going over the extremities of the bottom through stables, in the same manner that the body of a cart is bolted down to the shafts: a cable about fifty yards long was fastened at one end to the air balloon, and at the other to the stump of a large oak tree. The poor fellow had also contrived a valve, to which a cord was fixed, and by means of this he could introduce such a portion of atmospheric air, as to moderate the velocity with which the balloon ascended, or bring it gradually to the ground again.

[16] This letter was brought to my attention by Mr. R. T. H. Halsey of New Haven.

All these things being thus prepared, he went out two successive moon-light nights, attended only by two common labourers, whom he had sworn to secrecy.

The first night he sent the machine up twice to the length of the cable, first by itself, and then again after having suspended from it a large wooden chair, or rather box, loaded with three hundred weight of lead. All this went on extremely well, and both times he brought it down again very safe by the gradual admission of atmospheric air. I should have mentioned, what did him the greatest credit, his having a power, by a process of his own inventing, to supply the machine instantaneously after its descent with an ample provision of inflammable air for its next expedition.

The second night he set out again, as soon as he durst, for fear of intruders, ordering his two attendants to bring with them one of the horses used for towing barges along the Seine. His first trial this time, was to place himself in the chair instead of the leaden weight, and having gone up to the length of the cable, he brought himself safe down again by means of the air valve.

The success of this little flight having banished all distrust from his mind, he eagerly set about his grand experiment, which was no other than to fasten the cable round the horse's body, instead of the tree, that after moderating with his air valve, the tendency of the machine upwards, till it was overbalanced by the weight of the horse, he might be towed along at pleasure in this aerial vehicle. When he was properly seated, he ordered his two attendants once more to draw the bolts which fastened him to the earth, and up he flew—flew indeed! for, alas, at the very instant of his departure, an unfortunate puff of wind taking off his hat, he in snatching at it, let go the moderating string of the machine which, being full of inflammable air, rushed upwards with such impetuosity as to force not only him, but the horse too, into the upper regions of the atmosphere. The weather being clouded over, the machine and its appendages were soon out of sight. Messengers have been dispatched along every direction in which the wind has blown ever since

> that dreadful accident, and it is said that last night one of these returned with advice that the horse had fallen on the roof of a house at Fontainbleau, with about 20 yards of the cable which appeared to have given way; upon which report our philosophers assure us that the loss of so much weight, by forcing poor Fathom up a mile or two higher into air utterly unfit for respiration, must have put an immediate end to his existence. What makes this loss the more lamentable is, that this unfortunate but ingenious man was held in such esteem by his great master, that he is supposed to have had no small share in planning those dexterous manœuvres, by which his countrymen have been *raised* to that eminence among the nations, which makes all the world look up to them with equal delight and astonishment. Some superstitious or ill-minded persons affect to call the poor man's fate ominous, &c. but I forbear anticipating upon the many pasquinades to which this tragical event will no doubt give birth among the witty and the unfeeling.

Such is the whole document. What is its significance?

If this letter is to be believed, we may conclude that one Joseph Fathom, hitherto unknown, deserves a place in the annals of history and of science. According to the letter, he was experimenting with a balloon of the Charles type (that is, a balloon filled with hydrogen) rather than a Montgolfier type (filled with hot air kept warm by a small fire at the mouth of the bag). To control the "inflammable air," i.e. the hydrogen, Fathom had invented a valve mechanism which appears to have functioned perfectly. This invention is usually attributed to the inventor of the balloon, Charles himself, as we have stated above. The second claim to a place in the annals would be the fact that Fathom was the first casualty of the air, that is, he was the first person to die in an air-accident. Finally, he had, in the words of the letter, "no small share in planning those dexterous manœuvres, by which his countrymen have been raised to that eminence. . . ."

Although we are told that Joseph Fathom "was held in such esteem by his great master," "the celebrated Dr. F——," who is obviously Benjamin Franklin, and although we are

told further that he was Franklin's "American amanuensis," no mention of him is to be found in Franklin's correspondence, nor has any biographer of Franklin ever mentioned his name. Nor, for that matter, does any mention of this accident appear in Franklin's writings. Further suspicion concerning the validity of this account arises from the fact that I have been unable to find any corroboration of it in other English newspapers or in the French newspapers. And the very account of the accident itself seems not a little fantastic.

Burdened with the weight of a heavy horse, the balloon would hardly have rushed up with such great "impetuosity." Then too, if Fathom let go of the "moderating string," could he not regain it? And, one wonders, if the horse were found at Fontainbleau, why was not "poor Fathom" found? And, lastly, although the valve mechanism is perfectly feasible as means of controlling ascent and also descent (since it was actually used in that way by Charles), I cannot understand the meaning of the other part of his invention. The letter states that "what did him the greatest credit" was "his having a power, by a process of his own inventing, to supply the machine instantaneously after its descent with an ample provision of inflammable air for its next expedition." As mentioned at the beginning of this article, Charles filled his balloon by collecting hydrogen from the action of sulphuric acid on iron filings. This process was a slow one and it took him several days to fill his balloon. Three quarters of a century later, when Prof. Lowe was ballooning for the Federal army during the Civil War, an elaborate field mechanism was still necessary (See Fig. 11) and the process took several hours. Even today, with our great scientific and technical prowess, refilling a balloon is far from the "instantaneous" process which we are to believe was invented by Joseph Fathom.

If this is a hoax, one is naturally led to enquire what purpose it served. This is made clear by the last several sentences. The writer declares that he will not moralize about the event, but . . . ! He ends with the suggestion that one of the chief brains behind the American Revolution has met with a fate which "some superstitious or ill-minded

persons affect to call . . . ominous." The ulterior motive may then have been to show that disobedience to the Crown leads men to go even further on the road to perdition, that is to challenge the established order of nature herself. And the penalty for the latter is violent and horrible death.[17]

One sentence of this letter reads: ". . . Joseph Fathom, an American amanuensis to the celebrated Dr. F——, who has often had hair-breadth escapes from the dangers into which he has been betrayed by his eagerness in the imitation of his master's trials. . . ." This would indicate that Franklin had made "trials," or had actually made balloon ascensions. Not only is there no record of Franklin's having made experiments with balloons, but the extracts quoted at the early part of this article do not give positive indication that Franklin ever made a balloon ascension at all.

It seems most likely, then, that the letter is, after all, but one of those hoaxes in which the eighteenth century delighted. The name "Fathom" used in the letter may have been inspired by Tobias Smollet's novel, "The Adventures of Ferdinand Count Fathom" (*ed. princ.* London, 1753; ed. 2, 1771). One cannot tell whether or not this letter was ever brought to Franklin's attention. If it was, then it may have been the source of his remark to Banks, quoted above, wherein he expressed such regret that the English did not take the new invention seriously. This letter to Banks was written on Nov. 21, 1783, ten days after the date of the issue of *The General Evening Post* containing the hoax letter about "Fathom." But whether or not this is the case must remain, for the present, a curious little unsolved mystery.[18]

[17] One may note, in this connection, that according to the writer of the letter, Fathom "is supposed to have had no small share in planning those dexterous manœuvres, by which his countrymen have been *raised* to that eminence among the nations, which makes all the world look up to them with equal astonishment and delight." The word *raised* is italicised by the writer; it is the only word italicised in the whole body of the letter. This makes it appear that perhaps "those dexterous manœuvres" did not, in the mind of the writer, actually *raise* his countrymen to a genuinely high level. And the word "raised" is a pun on the manner of his "death."

[18] I should like to acknowledge the kindness of Mr. Carl Van Doren, with whom I have had the privilege of consulting during the course of preparing this article.

ON THE PROBLEM OF TEMPERATURE DISTRIBUTION IN PLANE PLATES.

BY

I. MALKIN,

Boston, Mass.

1. *Introduction.*—The solution of the problem of temperature distribution in thin-walled solids is of considerable interest in many technical questions, particularly as the basis for an attack of the problem of thermal stresses in important parts of heat power engines.

In the case of *steady* conditions it may be sometimes sufficient to approximate the solution of the first named problem by a temperature distribution which is linear across the wall of the solid. Such an approximation, occasionally occurring in the technical literature, can be used, however, neither in the case of temperature distributions *varying with the time*, nor in such characterized by considerable *non-uniformity of the temperature gradient* in the boundary surfaces even in steady conditions. Sharp changes in *curvature*, equally requring careful consideration in technical problems, affect the temperature distribution across the wall in general as well.

With these particular points in view an operational procedure in treating the problem will be developed below free of any approximations concerning the law of distribution across the wall of the solid. It is only assumed that the temperature of the boundary surface is free of discontinuities along the main (non-lateral) surfaces of the solid. Then the procedure yields a solution which satisfies the boundary conditions of the main surfaces; as to the lateral (narrow or small) boundary surfaces, the influence of their temperature distributions is either negligible as can be concluded from certain known solutions of the problem of conduction of heat in solids,[1] or their boundary conditions can be satisfied in

[1] Cf. H. S. Carslaw, "Introduction to the Mathematical Theory of the Conduction of Heat in Solids," London, 1921, p. 13 and p. 89; see also formula (*A*) of this paper.

using additional solutions, often available, of the standard types just mentioned.[1]

An illustration is given by the axisymmetrical case, represented in Fig. 2 below, of a temperature distribution in a circular plane plate of a comparatively small diameter. In using the solution developed in this paper we obtain in the example of Fig. 2 a temperature function which yields the required boundary distributions aA and cC resp. at the plane surfaces of the plate, and a certain (non-prescribed) temperature distribution adc at the cylindrical surface of the plate. If a linear (or any other arbitrary) distribution abc is required along the latter surface, a certain standard solution [see formula (A) below] must be superimposed on the first solution. The diagram Fig. 2 shows that the changes δ of the temperature distribution produced by the additional solution disappear rapidly in some distance from the cylindrical surface, provided that the maximum ordinate between the lines abc and adc does not exceed certain limits as compared with the maximum amount (see ordinate $a5$ in Fig. 2) of temperature differences occurring in the main boundary surfaces.

The latter remark concerning the influence of the lateral surfaces refers primarily to problems of *steady* temperature distributions. The influence of the lateral surfaces is, however, in general negligible also in the case of temperature distributions *varying with the time* due to surface heating. In problems of this kind the *beginning stage* of the heating process is especially important from the point of view of thermal stresses; in this stage the heat penetration and temperature rise is confined to a close vicinity of the heated surface; excluding, therefore, the immediate vicinity of the narrow or small (lateral) surfaces, and restricting ourselves to consideration of the remaining part of the solid, we will obtain, by means of our procedure, a practically very accurate solution of the problem of temperature distributions varying with the time due to surface heating.

As to the characteristics of the procedure itself, they can be stated briefly as follows.

The temperature distribution of the main surfaces of the solid are given, or approximated, by *finite polynomials* (in

terms of the coördinates measured in the boundary surfaces, and, in the problems of surface heating, also of the time coördinate), in order to simplify the purely analytical aspects of the problem. Then the solution is obtained, in the case of *steady* temperature distributions, again in the form of a finite expression convenient for computations.

The problem of temperature distributions, *varying with the time*, is restricted to the technically important case of surface heating. In this problem the solution appears in the form of a sum of two expressions, the first of which is analogous to the solution of the problem of steady temperatures, while the second is an infinite series of rapidly progressing negative powers of the basis e of natural logarithms. The convergence properties of this infinite series can be established with the aid of the elements of the theory of convergence of solutions obtained in the form of infinite series. Another procedure has been chosen, however, in order to avoid purely mathematical discussions in the case of the second portion of the solution as well. By means of a simple reduction indicated in the last sections of this paper and illustrated in an example, the infinite series is approximated, with sufficient accuracy, by a finite expression. In this manner the solution is always obtained in a finite form.

In concluding we may remark that the method offered will be of practical value under certain conditions of boundary temperature continuity referred to above; it will be found well adapted to the solution of the problem of temperature distributions in plane and in (sharply) bent plates [2] as well as in cylindrical shafts and shells, especially in such shells with bottom,[2] and in the problem of temperature distributions varying with the time.

An important practical advantage of the solution here offered is this. While the standard solutions are expressed in terms of quadratures and thus require evaluation of a sequence of generally complicated definite integrals, the solutions given in the present contribution are free of integrations which are all carried out in the general part of the procedure. This

[2] See I. Malkin, "Zum Einfluss scharfer Kruemmungsaenderungen auf die Temperaturverteilung in duennwandigen Koerpern," in the *Schweizer Archiv*, 1938, No. 12.

especially tends to simplify considerably the practical treatment of the thermoelastic problem of the solid.[3]

The present paper is an application of the method to the problem of steady and of variable tempeature distributions in *plane plates* under boundary conditions of considerable generality.

2. *General Integration in the Case of Steady Temperature Distributions.*—Consider a plane plate of thickness $2h$, referred to a rectangular Cartesian xyz- or a cylindrical $r\vartheta z$-system of coördinates, with $x = r\cos\vartheta$, $y = r\sin\vartheta$, the middle plane being given by $z = 0$. Designating by $\overline{T}$ the temperature of the plate and introducing the dimensionless quantities $\xi = x/h$, $\eta = y/h$, $\zeta = z/h$, $\rho = r/h$, $T = \overline{T}/T^*$, wherein T^* is a constant temperature, the differential equation of steady temperature distribution can be written in the dimensionless form

$$\Delta T \equiv T'' + DT = 0, \tag{1}$$

wherein the accents are used for differentiations with respect to ζ, while

$$D = \frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\vartheta^2}, \tag{2}$$

The surface conditions will be considered in the two following sections.

We shall use, as a solution of the problem expressed by (1), the expansion

$$\begin{aligned} T = {} & F_0T^0 + F_1DT^0 + F_2D^2T^0 + \cdots \\ & + G_0P^0 + G_1DP^0 + G_2D^2P^0 + \cdots, \end{aligned} \tag{3}$$

in which the quantities F_j, G_j, $j = 0, 1, 2, 3, \cdots$, are functions of ζ only, while T^0 and P^0 are functions (finite polynomials) of ξ and η, or of ρ and ϑ, derived, in a manner indicated below, from the boundary conditions of the temperature distribution. Substituting (3) into (1), we find the latter satisfied if the conditions

$$H_0'' = 0; \qquad H_{j+1}'' + H_j = 0 \qquad (j = 0, 1, 2, 3, \cdots) \tag{4}$$

are fulfilled for $H_j = F_j$ as well as for $H_j = G_j$.

[3] See A. E. H. Love, "Theory of Elasticity," 4th ed., Cambridge (Great Britain), 1927, pp. 108–109.

From the first of the equations (4) we find that

$$F_0(\zeta) = 1 \qquad (5) \qquad\qquad G_0(\zeta) = \zeta \qquad (6)$$

may be assumed. The further functions F_j, G_j, $j = 1, 2, 3, \cdots$, and the functions T^0, P^0 will be determined in the subsequent sections in accordance with the boundary conditions of the types considered.

3. *The Case of Boundary Surface Temperatures Being Given.*—Introducing (5) and (6) into the second of the equations (4), taken for $j = 0$, and imposing on the functions F_1 and G_1 the condition that they disappear at both of the plane surfaces $\zeta = \pm 1$, we obtain $F_1 = \frac{1}{2}(1 - \zeta^2)$, $G_1 = \frac{1}{6}\zeta(1 - \zeta^2)$. Starting now from

$$F_1 = \tfrac{1}{2}(1 - \zeta^2) = 2\left(\frac{1}{\pi}\right)^3 \sum_{\lambda=0}^{\infty} (-1)^\lambda \frac{1}{(\lambda + \frac{1}{2})^3} \cos(\lambda + \tfrac{1}{2})\pi\zeta, \qquad (7)$$

$$G_1 = \tfrac{1}{6}(\zeta - \zeta^3) = 2\left(\frac{1}{\pi}\right)^3 \sum_{\lambda=0}^{\infty} (-1)^\lambda \frac{1}{(\lambda + 1)^3} \sin(\lambda + 1)\pi\zeta, \qquad (8)$$

and introducing the Fourier expansions (7), (8) into the second of the equations (4), we find by successive integration, again under the condition that the functions F_j, G_j, $j = 1, 2, 3, \cdots$, vanish at the boundary surfaces $\zeta = \pm 1$,

$$F_j = 2\left(\frac{1}{\pi}\right)^{2j+1} \sum_{\lambda=0}^{\infty} (-1)^\lambda \frac{1}{(\lambda + \frac{1}{2})^{2j+1}} \cos(\lambda + \tfrac{1}{2})\pi\zeta \qquad (9)$$

$$G_j = 2\left(\frac{1}{\pi}\right)^{2j+1} \sum_{\lambda=0}^{\infty} (-1)^\lambda \frac{1}{(\lambda + 1)^{2j+1}} \sin(\lambda + 1)\pi\zeta; \qquad (10)$$

$$F_0 = 1; \quad G_0 = \zeta; \quad (j = 1, 2, 3, \cdots) \quad (-1 \leqslant \zeta \leqslant 1). \qquad (11)$$

The functions (9), (10), (11) are represented graphically in Fig. 1.

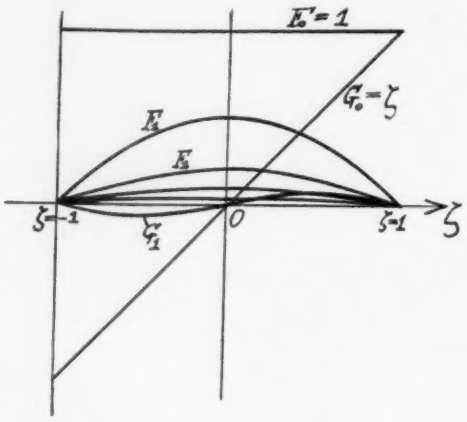
As to the functions T^0 and P^0 we define them by

$$T^0 = \tfrac{1}{2}(T_{\zeta=1} + T_{\zeta=-1}), \qquad P^0 = \tfrac{1}{2}(T_{\zeta=1} - T_{\zeta=-1}), \qquad (12)$$

wherein both of the functions $T_{\zeta=1}$ and $T_{\zeta=-1}$ are to be considered as given *finite polynomials* in terms of ξ, η.

The expansion (3) has consequently a finite number of terms. It represents, in connection with the definitions (12) and (9), (10), (11), the solution of our problem of steady temperature distribution in plates with given boundary

FIG. 1.



temperature distributions of the main surfaces; as to the lateral surfaces, their influence is illustrated by the example at the end of 3.

The verification of the solution does not present any difficulties.

For small values of the index j the representations

$$F_0 = 1; \qquad F_1 = \tfrac{1}{2}(1 - \zeta^2);$$

$$F_2 = \frac{5}{24} - \frac{1}{4}\zeta^2 + \frac{1}{24}\zeta^4; \qquad \cdots \tag{13}$$

$$G_0 = \zeta; \qquad G_1 = \tfrac{1}{6}(\zeta - \zeta^3);$$

$$G_2 = \frac{7}{360}\zeta - \frac{1}{36}\zeta^3 + \frac{1}{120}\zeta^5; \qquad \cdots \tag{14}$$

may be used. For larger values of j the formulæ (9), (10) are more convenient because of their rapid convergence. In practical applications it will be useful to have the functions F_j, G_j, $j = 1, 2, 3, \cdots$, tabulated once for all.

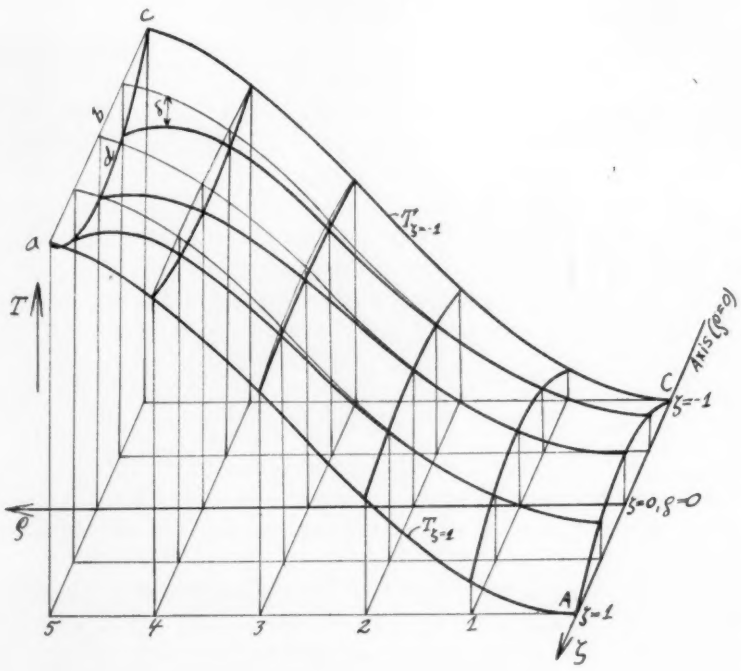
Take as an example the case of a circular plate, of thickness $2h$ and diameter $2r_1$, in axisymmetrical conditions determined by the boundary temperature distributions

$$T_{\zeta=1} = T_{\zeta=-1} = T^0 = 2\rho_1^2\rho^2 - \rho^4; \qquad P^0 = 0;$$
$$(0 \leqslant \rho = r/h \leqslant r_1/h = \rho_1 = 5).$$

The solution is, on the basis of (3), (12), (13), (14) given by

$$T = 50\rho^2 - \rho^4 + 8(25 - 2\rho^2)F_1 - 64F_2 \qquad (15)$$

FIG. 2.



and graphically represented by Fig. 2 (bold lines). If a definite axisymmetrical temperature distribution is prescribed for the cylindrical surface of the plate, an auxiliary solution must be superimposed on the solution (15); this auxiliary

solution is, in its general form,

$$T' = \sum_{j=1,2,3,\cdots} \left\{ \left(\alpha_j \sin j\frac{\pi}{2}\zeta + \beta_j \cos j\frac{\pi}{2}\zeta \right) \frac{J_0\left(ij\frac{\pi}{2}\rho\right)}{J_0\left(ij\frac{\pi}{2}\rho_1\right)} \right\}, \quad (A)$$

wherein J_0 is the Bessel function of the first kind of the order zero and $i^2 = -1$, while the constants α_j and β_j are coefficients of the Fourier expansion of the temperature distribution $T_{\rho=\rho_1}$ of the cylindrical surface.[4]

In applying formula (A) to our example supplemented by the condition $T_{\rho=\rho_1} = \text{const.}$ (straight line *abc* in Fig. 2), the additional solution assumes practically the form

$$T' = \beta \frac{J_0\left(i\frac{\pi}{2}\rho\right)}{J_0\left(i\frac{\pi}{2}\rho_1\right)} \cos \tfrac{1}{2}\pi\zeta, \quad (15a)$$

because the curve *adc* in Fig. 2 can be approximated by const. $\cos \frac{1}{2}\pi\zeta$. The result of superposition is represented by the thin curves in Fig. 2 (cf. Introduction).

4. *General Boundary Conditions.*—We consider now boundary conditions of the well known general form

$$\begin{aligned} [q(\partial T/\partial\zeta) + T]_{\zeta=1} &= \theta_{\zeta=1}, \\ [-q(\partial T/\partial\zeta) + T]_{\zeta=-1} &= \theta_{\zeta=-1}, \end{aligned} \quad (16)$$

wherein q is again a dimensionless constant $(0 \leqslant q \leqslant \infty)$, which we assume to have the same value on both sides of the plate, while θ designates the temperature of the adjacent medium.

Starting again from (5) and (6) and introducing each of these two quantities into the second of the equations (4), taken for $j = 0$, we again integrate, but now under the boundary conditions (16) taken in *homogeneous* form, or

$$\begin{aligned} [q(\partial H/\partial\zeta) + H]_{\zeta=1} &= 0, \\ [-q(\partial H/\partial\zeta) + H]_{\zeta=-1} &= 0, \end{aligned} \quad (17)$$

[4] The formula (A), not found in Carslaw's textbook is, however, easily verified in the case of a finite number of terms, and this is the only case in which we are practically interested here.

when $H = F_1$ and when $H = G_1$, and this leads to

$$F_1 = -\tfrac{1}{2}\zeta^2 + \tfrac{1}{2} + q, \tag{18}$$

$$G_1 = \frac{1}{6}\left(\frac{1+3q}{1+q}\zeta - \zeta^3\right). \tag{19}$$

We expand the two functions (18), (19) again into Fourier series, with the component functions

$$\cos(n_{2s+1}\zeta), \tag{20}$$

$$\sin(n_{2s+2}\zeta), \tag{21}$$

$$(s = 0, 1, 2, 3, \cdots).$$

wherein the constants n_{2s+1} and n_{2s+2} are obtained by substituting each of the functions (20), (21) into the boundary conditions (17), which gives the well known equations [5]

$$\operatorname{ctg} n_{2s+1} = n_{2s+1}q, \tag{22}$$

$$\operatorname{tg} n_{2s+2} = -n_{2s+2}q \tag{23}$$

for those constants; the roots of these two equations are reproduced in the Tables 1 and 2 resp. Thus each of the functions (20), (21) satisfies both of the boundary conditions (17).

Expanding now the functions F_1 and G_1, as given by (18), (19), into Fourier series with the component functions (20), (21) resp., with the aid of the formulæ [6]

$$N_{2s+1} = \frac{\int_{-1}^{1} F_1 \cos(n_{2s+1}\zeta)d\zeta}{\int_{-1}^{1} \cos^2(n_{2s+1}\zeta)d\zeta}, \qquad N_{2s+2} = \frac{\int_{-1}^{1} G_1 \sin(n_{2s+2}\zeta)d\zeta}{\int_{-1}^{1} \sin^2(n_{2s+2}\zeta)d\zeta}$$

for the constant coefficients of the expansion, we find the infinite series

$$F_1 = 2\left[\frac{1}{n_1^3}C_1(q)\cos n_1\zeta + \frac{1}{n_3^3}C_3(q)\cos n_3\zeta + \cdots\right], \tag{24}$$

$$G_1 = -2(1+q)\left[\frac{1}{n_2^3}C_2(q)\sin n_2\zeta + \frac{1}{n_4^3}C_4(q)\sin n_4\zeta + \cdots\right], \tag{25}$$

[5] See f. i. H. Groeber und S. Erk, "Die Grundgesetze der Waermeuebertragung, Berlin, 1933, pp. 17–18. The Tables 1 and 2 are taken from this book.

[6] See H. Groeber und S. Erk, l.c., p. 32.

wherein

$$C_{2s+1}(q) = \frac{\sin n_{2s+1}}{1 + \dfrac{\sin 2n_{2s+1}}{2n_{2s+1}}} \; ; \quad s\pi \leqslant n_{2s+1} \leqslant (s + \tfrac{1}{2})\pi \qquad (24a)$$

$$(s = 0, 1, 2, \cdots),$$

$$C_{2s+2}(q) = \frac{\cos n_{2s+2}}{1 - \dfrac{\sin 2n_{2s+2}}{2n_{2s+2}}} \; ;$$

$$(s + \tfrac{1}{2})\pi \leqslant n_{2s+2} \leqslant (s + 1)\pi. \qquad (25a)$$

The quantities (24*a*), (25*a*) are tabulated and graphically represented in the following section 5, see Fig. 3; a graphical

TABLE 1.

Roots of the First of the Equations (20).

$1/q$	n_1	n_3	n_5	n_7	n_9
∞	0.5π	1.5π	2.5π	3.5π	4.5π
1000	1.57	4.71	7.84	10.98	14.13
100	1.56	4.66	7.77	10.88	14.00
50	1.54	4.62	7.70	10.78	13.87
20	1.50	4.49	7.49	10.51	13.55
10	1.43	4.30	7.22	10.20	13.22
4.0	1.26	3.93	6.81	9.78	12.87
1.0	0.86	3.42	6.43	9.52	12.65
0.5	0.65	3.29	6.36	9.47	12.61
0.1	0.31	3.17	6.30	9.43	12.57
0.01	0.10	3.14	6.28	9.42	12.57
0	0	π	2π	3π	4π

TABLE 2.

Roots of the Second of the Equations (20).

$1/q$	n_0	n_2	n_4	n_6	n_8	n_{10}
∞	0	π	2π	3π	4π	5π
1000	0	3.14	6.27	9.41	12.56	15.69
100	0	3.11	6.22	9.33	12.45	15.56
50	0	3.08	6.16	9.24	12.33	15.41
20	0	2.99	5.99	9.00	12.04	15.07
10	0	2.86	5.76	8.70	11.71	14.74
4.0	0	2.57	5.35	8.30	11.34	14.41
1.0	0	2.03	4.91	7.98	11.09	14.21
0.5	0	1.84	4.80	7.91	11.05	14.18
0.1	0	1.63	4.73	7.86	11.01	14.15
0.01	0	1.58	4.71	7.85	11.00	14.15
0	0	0.5π	1.5π	2.5π	3.5π	4.5π

representation of the expansions (24), (25) for a special value of the constant q is found in the same section 5, see Fig. 4.

Substituting each of the two expansions (24), (25) again into the second of the differential equations (4), we obtain by successive integration, which is permitted by virtue of the criterion of uniform convergence, the expansions

$$\left.\begin{aligned} F_j = 2\Big[&\frac{1}{{n_1}^{2j+1}} C_1(q) \cos n_1\zeta \\ &+ \frac{1}{{n_3}^{2j+1}} C_3(q) \cos n_3\zeta + \cdots\Big] \end{aligned}\right\} (j = 1, 2, 3, \cdots) \quad (26)$$

$$\left.\begin{aligned} G_j = -2(1+q)\Big[&\frac{1}{{n_2}^{2j+1}} C_2(q) \sin n_2\zeta \\ &+ \frac{1}{{n_4}^{2j+1}} C_4(q) \sin n_4\zeta + \cdots\Big] \end{aligned}\right\} \quad (27)$$

$$F_0 = 1; \qquad G_0 = \zeta; \qquad (-1 \leqslant \zeta \leqslant 1). \quad (28)$$

Each of the expansions (26), (27) satisfies both of the boundary conditions (17), as is to be concluded again from the uniform convergence, in the entire closed interval $-1 \leqslant \zeta \leqslant 1$, of all of the infinite series involved.

The expansions (26), (27) are generalizations of those given in (9), (10); the latter can be obtained from the former by substituting $q = 0$. In the other extreme case, $q = \infty$ (plate material impervious to heat), the solution is obviously $T = \text{const.}$ throughout the plate.

As to the functions T^0 and P^0, they are defined now by

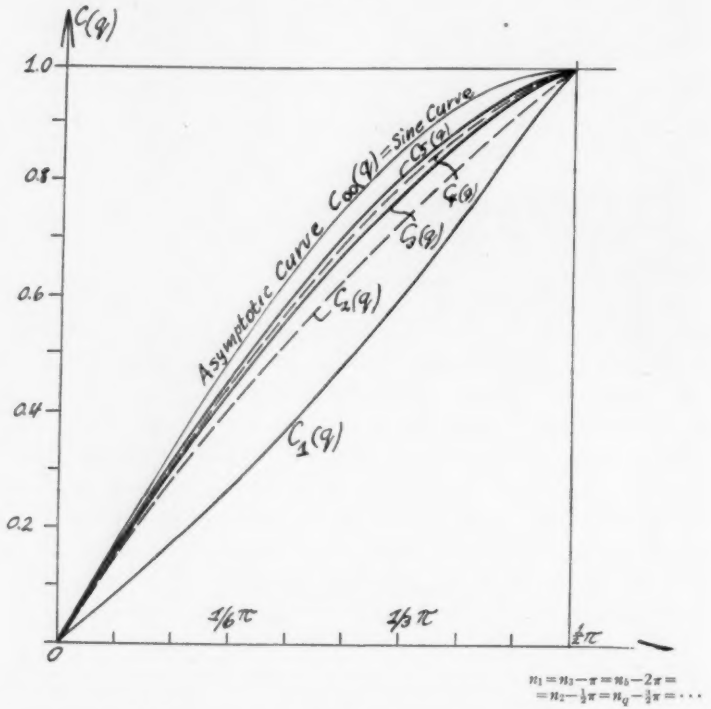
$$T^0 = \tfrac{1}{2}(\theta_{\zeta=1} + \theta_{\zeta=-1}), \qquad P^0 = \frac{1}{2(1+q)}(\theta_{\zeta=1} - \theta_{\zeta=-1}), \quad (29)$$

wherein the boundary surface temperatures $\theta_{\zeta=1}$ and $\theta_{\zeta=-1}$ of the adjacent medium are again to be considered as given *finite polynomials* in terms of ξ, η so that the expansion (3) again consists of a *finite* number of terms. This expansion (3) represents then, in conjunction with the definitions (29), (26), (27), (28), and with the symbol D as given in (2), the solution of our problem of steady temperature distributions in plane plates with boundary conditions of the form (16) for

the main surfaces of the plate. The verification of this solution is entirely analogous to that of the previous case.

5. *Remarks Concerning the Coefficients $C(q)$ and Illustration of the Expansions* (24), (25).—The coefficients $C(q)$, given in the formulæ (24*a*), (25*a*), are computed in the following table 3 and graphically represented in Fig. 3.

FIG. 3.



The practical use of the Tables 1, 2, 3 is illustrated in the following determination of the expansions (26), (27) for the special case $q = 1$.

For $q = 1$ we obtain from the Tables 1 and 2

$$n_1 = 0.86; \quad n_2 = 2.03; \quad n_3 = 3.42; \quad n_4 = 4.91; \quad n_5 = 6.43.$$

The corresponding values of the constant $C(q)$, as obtained from Table 3 by interpolation, or from Fig. 3, drawn to a

larger scale, are

$$C_1 = 0.482; \quad C_2 = -0.369; \quad C_3 = -0.255;$$
$$C_4 = 0.188; \quad C_5 = 0.143;$$

substituting the value of q into (18), (19), and those of the

TABLE 3.

Numerical Values of the Coefficients (24a), (25a).

n_1	C_1	n_3	C_3	n_5	C_5	n_2	C_2	n_4	C_4
0.000	0.000	3.142	0.000	6.283	0.000	1.571	0.000	4.712	0.000
0.131	0.066	3.272	−0.126	6.414	0.128	1.702	−0.121	4.843	0.127
0.262	0.132	3.403	−0.241	6.545	0.249	1.833	−0.228	4.974	0.246
0.393	0.201	3.534	−0.348	6.676	0.363	1.964	−0.324	5.105	0.358
0.524	0.274	3.665	−0.447	6.807	0.470	2.094	−0.414	5.236	0.462
0.654	0.350	3.796	−0.540	6.938	0.569	2.225	−0.500	5.367	0.559
0.785	0.432	3.927	−0.627	7.069	0.660	2.356	−0.583	5.498	0.648
0.916	0.520	4.058	−0.709	7.200	0.743	2.487	−0.664	5.629	0.731
1.047	0.613	4.189	−0.785	7.330	0.818	2.618	−0.743	5.760	0.805
1.178	0.711	4.320	−0.854	7.461	0.882	2.749	−0.819	5.890	0.872
1.309	0.811	4.451	−0.915	7.592	0.935	2.880	−0.889	6.021	0.927
1.440	0.910	4.581	−0.964	7.723	0.975	3.011	−0.951	6.152	0.971
1.571	1.000	4.712	−1.000	7.854	1.000	3.142	−1.000	6.283	1.000

constants $C_1(q)$, $C_2(q)$, $\cdots$, n_1, n_2, $\cdots$, into (24), (25), we get

$$F_1 = \tfrac{1}{2}(3 - \zeta^2) = 1.516 \cos 0.86\zeta - 0.013 \cos 3.42\zeta + - \cdots, \quad (30)$$

$$G_1 = \tfrac{1}{6}(2\zeta - \zeta^3) = 0.176 \sin 2.03\zeta - 0.006 \sin 4.91\zeta + - \cdots. \quad (31)$$

The two double formulæ (30), (31) are graphically visualized in Fig. 4. Note that in each of the two expansions (30), (31) the first term is practically entirely sufficient to represent the function in question. This is all the more the case with the functions F_j, G_j, $j > 1$, because of the factors $n^{-(2j+1)}$.

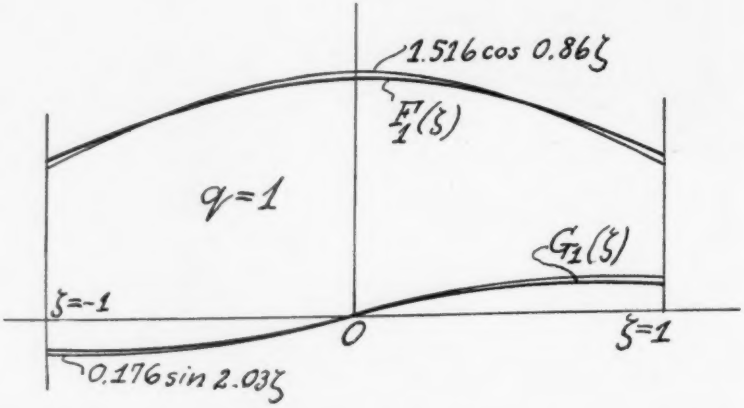
If $n_1 < 1$, the coefficient of the first term in the expansions (26) will increase with progressing j [in consequence of the circumstance that the point $q = \infty$, $n_1 = 0$ is a singular point of the problem, see section 4, remark preceding formula (29)]; this behavior of the first term in F_j, and thus of F_j

itself, does not create difficulties, however, the number of terms in (3) being finite.

Thus we have with sufficient accuracy, in the case $q = 1$ considered here, at once

$$\left.\begin{aligned} F_2 &= \frac{1.516}{0.86^2}\cos 0.86\zeta = 2.05\cos 0.86\zeta; \\ F_3 &= \frac{1.516}{0.86^4}\cos 0.86\zeta = 2.77\cos 0.86\zeta; \quad \cdots \\ G_2 &= \frac{0.176}{2.03^2}\sin 2.03\zeta = 0.043\sin 2.03\zeta; \\ G_3 &= \frac{0.176}{2.03^4}\sin 2.03\zeta = 0.010\sin 2.03\zeta; \quad \cdots \end{aligned}\right\} \quad (32)$$

FIG. 4.



Take again as an example the case of a circular plate of thickness $2h$ and diameter $2r_1$ in an axisymmetrical state determined by the boundary conditions of the form (16), in which

$$q = 1, \qquad \theta_{\zeta=1} = \theta_{\zeta=-1} = 2\rho_1^2\rho^2 - \rho^4,$$
$$0 \leqslant \rho = r/h \leqslant r_1/h = \rho_1 = 5$$

(cf. the example treated in section 3).

According to (29) we will have again, as in the first example mentioned,

$$T^0 = 2\rho_1^2\rho^2 - \rho^4, \qquad P^0 = 0$$

and with the aid of the formulæ (32) the solution (3) assumes, for $\rho_1 = 5$, the simple form

$$T = 50\rho^2 - \rho^4 + (172 - 24.3\rho^2) \cos 0.86\zeta. \qquad (33)$$

This (approximate) solution and the boundary temperatures $\theta_{\zeta=1}$ and $\theta_{\zeta=-1}$ of the adjacent medium are graphically represented in Fig. 5 (bold and broken curves resp.).

As to the question of a supplementary solution to be superimposed on (33) for the purpose of satisfying a boundary condition of the *cylindrical* surface of the plate, we restrict ourselves to the case when, in addition to the boundary conditions (16) of the plane surfaces, a certain temperature distribution is required at the boundary $\rho = \rho_1$, satisfying itself the conditions (16). Then the *additional* solution T' must satisfy the conditions (17) at the boundary $\rho = \rho_1$. Starting now from a finite sum

$$T'_{(\rho=\rho_1)} = A_1 \cos n_1\zeta + A_2 \sin n_2\zeta + A_3 \cos n_3\zeta + A_4 \sin n_4\zeta + \cdots,$$

in which the coefficients A are arbitrary constants, while the constants n are the same as before, we will obtain, in analogy to (A),

$$T' = \sum_s \left[A_{2s+1} \cos n_{2s+1}\zeta \frac{J_0(n_{2s+1}i\rho)}{J_0(n_{2s+1}i\rho_1)} + A_{2s+2} \sin n_{2s+2}\zeta \frac{J_0(n_{2s+2}i\rho)}{J_0(n_{2s+2}i\rho_1)} \right]. \qquad (B)$$

If the additionally prescribed temperature distribution at the surface $\rho = \rho_1$ does not satisfy the conditions (16), then an approximate solution can be obtained as shown in the following example.

Suppose, a constant temperature T_1 is prescribed for the surface $\rho = \rho_1$. The difference between T_1 and the temperature distribution (33) taken for $\rho = \rho_1 = 5$ is

$$T_I = T_1 - (50\rho_1^2 - \rho_1^4) - (172 - 24.3\rho_1^2) \cos 0.86\zeta = T_1 - 625 + 435 \cos 0.86\zeta. \qquad (34)$$

In using now the n- and C-values computed above we easily

obtain the (generalized) Fourier expansion

$$1 = 2\left(\frac{C_1}{n_1}\cos n_1\zeta + \frac{C_3}{n_3}\cos n_3\zeta + \frac{C_5}{n_5}\cos n_5\zeta + \cdots\right)$$
$$= 1.120\cos 0.86\zeta - 0.149\cos 3.42\zeta + 0.045\cos 6.43\zeta - + \cdots.$$

Substitution into (34) gives, with approximation,

$$T_I = [1.120(T_1 - 625) + 435]\cos 0.86\zeta - 0.149(T_1 - 625)\cos 3.42\zeta + 0.045(T_1 - 625)\cos 6.43\zeta.$$

This expression fulfils the boundary conditions (17), and the additional solution is, therefore, by virtue of (*B*), with approximation

$$\left.\begin{aligned} T' = {} & [1.120(T_1 - 625) + 435]\frac{J_0(0.86i\rho)}{J_0(0.86i\rho_1)}\cos 0.86\zeta \\ & - 0.149(T_1 - 625)\frac{J_0(3.42i\rho)}{J_0(3.42i\rho_1)}\cos 3.42\zeta \\ & + 0.045(T_1 - 625)\frac{J_0(6.43i\rho)}{J_0(6.43i\rho_1)}\cos 6.43\zeta - + \cdots. \end{aligned}\right\} \quad (35)$$

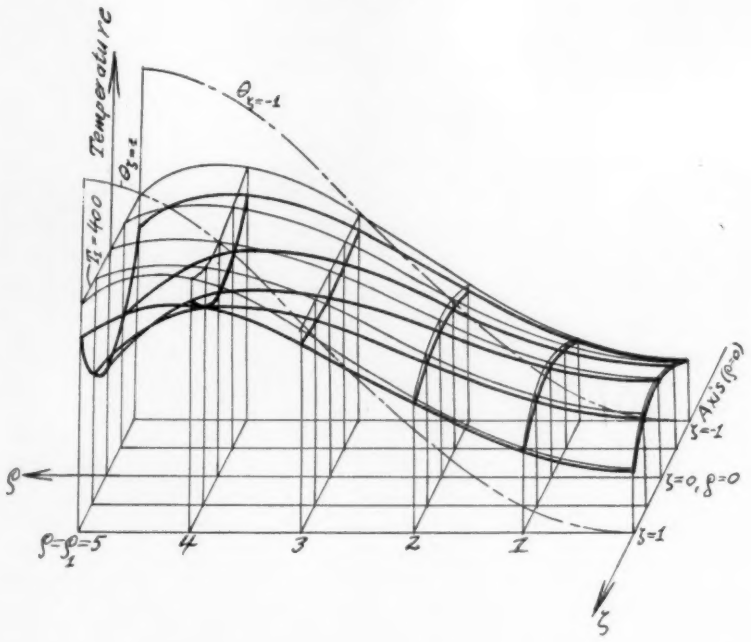
Taking f.i. $T_1 = 400$ and superimposing (35) on (33) we obtain the solution represented by the thin curves in Fig. 5. The reader will easily verify that the sum of (33) and (35) satisfies the boundary conditions (16) at the plane surfaces $\zeta = \pm 1$. It should not be forgotten, however, that (33) is an *approximate* expression, which does not satisfy the differential equation (1). Whenever derivatives of the temperature distribution of the plate are required, the additional solution (35) is to be taken in conjunction with the exact expression (3), from which (33) has been derived.

The procedure developed in the preceding sections shall be extended now, under certain modifications, to the problem of temperature distributions varying with the time.

6. *Variable Temperature Distributions; General Expression for Temperature Distributions in Conditions of Surface Heating.*—The following is a generalization of the procedure, outlined in the sections 2–5 of this paper, for the case of

temperature distributions varying with the time, in plane plates heated at the large surfaces. The only essential modification of the method is the change caused by the *initial condition*, which we assume to be $T = 0$ throughout the plate at the beginning of the heating process. We will see that this additional condition can be satisfied in a very simple

FIG. 5.



manner by superimposing, on an expression analogous to (3), an auxiliary solution, equally based upon the operational expansion used in the preceding sections of this paper.

With the aid of the same designations as above and of a new dimensionless symbol

$$\tau = \frac{k}{h^2} t, \tag{36}$$

wherein t is the time, and k ther thermal conductivity, we may write the differential equation of the temperature dis-

tribution varying with the time in the form

$$\Delta T - \frac{\partial T}{\partial \tau} = T'' + DT - \frac{\partial T}{\partial \tau} = 0. \qquad (1')$$

Furthermore we define, with reference to (2), a differential operator D_τ by the formula

$$D_\tau = D - \frac{\partial}{\partial \tau}. \qquad (2a)$$

Now consider the expression, analogous to (3),

$$T = \sum_j (F_j D_\tau{}^j T^0 + G_j D_\tau{}^j P^0), \qquad (3')$$

wherein T^0 and P^0 are functions of ξ, η, and τ, or of ρ, ϑ, and τ, derived, in the manner indicated by (12) or by (29) respectively, from the boundary conditions of the heating process, and $D_\tau{}^0 T^0 = T^0$, $D_\tau{}^0 P^0 = P^0$. It is easily seen that the expansion (3′) satisfies the equation (1′), but it will not satisfy, in general, the initial condition $T = 0$ throughout the plate when $\tau = 0$. To satisfy *all* conditions of the problem, we shall use, instead of (3′), the expression

$$T = \sum_j (F_j D_\tau{}^j T^0 + G_j D_\tau{}^j P^0) - T', \qquad (3'')$$

wherein T' is the auxiliary solution mentioned above. This additional solution is given explicitly in the subsequent sections 7 and 8.

7. *The Additional Solution in the Case of Given Boundary Surface Temperatures.*—The supplementary solution T' must obviously satisfy the boundary conditions

$$T' = 0 \quad \text{(at any time) when } \zeta = \pm 1 \qquad (37)$$

and the initial condition

$$T_i' = \left[\sum_j (F_j D_\tau{}^j T^0 + G_j D_\tau{}^j P^0)\right]_i, \qquad (38)$$

wherein the index i indicates the initial state at $\tau = 0$.

If T^0 and P^0 are *finite* polynomials, as we always assume in this paper, then the number of terms in the right side member of (38) is finite, too. If, in these conditions, the initial temperature is to be approximated within the limits

of a prescribed error ϵ_0, it is always possible to determine such a number N_0, that the error of approximation never exceeds the amount ϵ_0 in the entire interval $-1 \leqslant \zeta \leqslant 1$, if only all of the functions F_j, G_j are approximated by the first N_0 terms of their respective Fourier expansions. In using such approximations for the functions F_j, G_j instead of the functions themselves we avoid the necessity of certain purely analytical discussions.

Thus substituting for the functions F_j, G_j in (38) the first N_0 terms of their respective expansions as given in (9) and (10), we obtain, by means of a rearrangement of the terms, the finite expression

$$T_i' = W_{1i}\cos\tfrac{1}{2}\pi\zeta + W_{3i}\cos\tfrac{3}{2}\pi\zeta + \cdots + W_{2i}\sin\pi\zeta + W_{4i}\sin 2\pi\zeta + \cdots, \quad (39)$$

wherein

$$W_{2s+1} = (-1)^s 2(a_s{}^3D_\tau T^0 + a_s{}^5D_\tau{}^2T^0 + a_s{}^7D_\tau{}^3T^0 + \cdots);$$

$$a_s = \frac{2}{\pi(2s+1)}; \qquad (s = 0, 1, 2, 3, \cdots),$$

$$W_{2s} = (-1)^{s-1}2(b_s{}^3D_\tau P^0 + b_s{}^5D_\tau{}^2P^0 + b_s{}^7D_\tau{}^3P^0 + \cdots);$$

$$b_s = \frac{1}{\pi s}; \qquad (s = 1, 2, 3, 4, \cdots),$$

the functions W_{2s+1} and W_{2s} being finite polynomials in terms of ξ, η, and τ.

Having approximated the expansion (38) by the expression (39), which represents a Fourier series of a *finite* number of terms (with variable coefficients), we obtain the solution of the additional problem, stated above, in the form of a finite sum and with N as a finite number:

$$\left.\begin{aligned} T' = {} & \left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{1i}\right]e^{-(\pi^2/4)\tau}\cos\tfrac{1}{2}\pi\zeta \\ & + \left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{3i}\right]e^{-(9\pi^2/4)\tau}\cos\tfrac{3}{2}\pi\zeta + \cdots \\ & + \left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{2i}\right]e^{-\pi^2\tau}\sin\pi\zeta \\ & + \left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{4i}\right]e^{-4\pi^2\tau}\sin 2\pi\zeta + \cdots \end{aligned}\right\}. \quad (40)$$

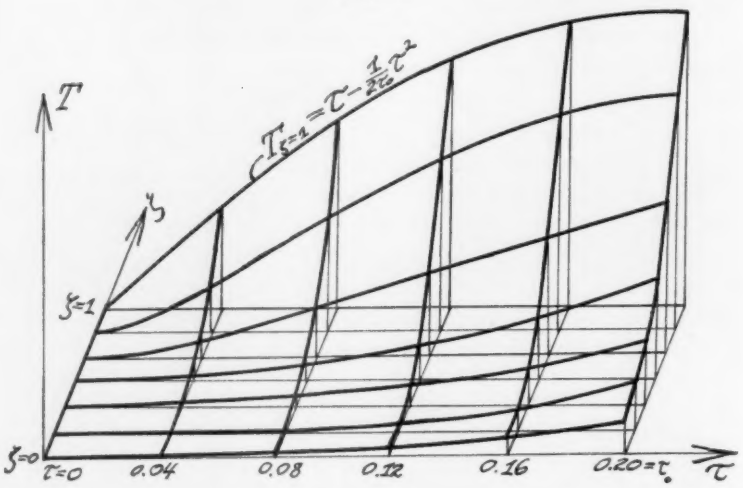
The verification of (40) and thus of (3″) is again very simple.

Take as an example the case of an *infinite* plane plate, of the thickness $2h$, heated at both of its surfaces $\zeta = \pm 1$ according to the law

$$T_{\zeta=1} = T_{\zeta=-1} = T^0 = \tau - \frac{1}{2\tau_0}\tau^2, \qquad P^0 = 0,$$

wherein τ_0 is a constant corresponding to the instant when the maximum temperature of the surface is reached, see Fig. 6.

FIG. 6.



The solution will be obviously symmetrical with respect to the middle plane $\zeta = 0$ (cf. the above example illustrated by Fig. 2); due to this symmetry the temperature distribution in question will also represent the solution of the problem of a plane plate of the thickness h heated at the surface $\zeta = 1$, and impervious to heat at the surface $\zeta = 0$ ($\partial T/\partial\zeta = 0$ when $\zeta = 0$).

The solution is

$$T = \tau - \frac{1}{2\tau_0}\tau^2 + \frac{16}{\pi^3}\left[\frac{\tau}{\tau_0} + \left(1 + \frac{4}{\pi^2\tau_0}\right)(e^{-(\pi^2/4)\tau} - 1)\right]\cos\tfrac{1}{2}\pi\zeta - \frac{16}{(3\pi)^3}\left\{\frac{\tau}{\tau_0} + \left[1 + \left(\frac{2}{3\pi}\right)^2\frac{1}{\tau_0}\right](e^{-(9\pi^2/4)\tau} - 1)\right\}\cos\tfrac{3}{2}\pi\zeta + \cdots.$$

Substituting f.i. $h = 1$ inch, $k = 0.02$ in^2/sec and assuming τ_0 to correspond to a time interval equal to 10 sec., so that $\tau_0 = (0.02/1^2) \times 10 = 0.2$, we will obtain, with practically sufficient accuracy,

$$T = \tau - 2.5\tau^2 + 0.516[5\tau + 3.03(e^{-(\pi^2/4)\tau} - 1)]\cos \tfrac{1}{2}\pi\zeta - 0.019[5\tau + 1.225(e^{-(9\pi^2/4)\tau} - 1)]\cos \tfrac{3}{2}\pi\zeta.$$

This solution is represented graphically in Fig. 6.

8. *The Additional Solution T' in the Case of General Boundary Conditions.*—In the case of boundary conditions of the form (16) the additional solution T' in the formula (3″) is obtained in a manner entirely analogous to that outlined in the preceding section 7. Namely, the additional solution T' must be now such a solution of (1′), which satisfies the *homogeneous* boundary conditions (17) and the initial condition

$$\left.\begin{aligned} T' &= \textstyle\sum_j (F_j D_\tau{}^j T^0 + G_j D_\tau{}^j P^0) \\ &\qquad\qquad \text{throughout the plate} \end{aligned}\right\} \text{when } \tau = 0. \quad (41)$$

$$\theta = 0 \qquad \text{at the surfaces } \zeta = \pm 1 \quad (42)$$

The condition (42) must be, of course, fulfilled by the given surface temperature distributions $\theta_{\zeta=1}$ and $\theta_{\zeta=-1}$ of the adjacent medium.

Assuming again the functions $\theta_{\zeta=1}$ and $\theta_{\zeta=-1}$, and thus also the functions T^0 and P^0 in (29), to be finite polynomials, and in using again finite approximations for the functions F_j, G_j, $j = 1, 2, 3, \cdots$, we replace the expression (41) by the generalized Fourier series

$$\left.\begin{aligned} T' = {} & W_{1i}e^{-n_1{}^2\tau}\cos n_1\zeta + W_{3i}e^{-n_3{}^2\tau}\cos n_3\zeta + \cdots \\ & + W_{2i}e^{-n_2{}^2\tau}\sin n_2\zeta + W_{4i}e^{-n_4{}^2\tau}\sin n_4\zeta + \cdots \end{aligned}\right\}, \quad (43)$$

wherein the index i refers to the initial state $\tau = 0$, and

$$\left.\begin{aligned} W_{2s+1} &= 2C_{2s+1}(q)\left[\frac{1}{n^3{}_{2s+1}} D_\tau T^0 + \frac{1}{n^5{}_{2s+1}} D_\tau{}^2 T^0 + \cdots\right] \\ &\qquad\qquad (s = 0, 1, 2, 3, \cdots) \\ W_{2s} &= -2(1+q)C_{2s}(q)\left[\frac{1}{n_{2s}{}^3} D_\tau P^0 + \frac{1}{n_{2s}{}^5} D_\tau{}^2 P^0 + \cdots\right] \\ &\qquad\qquad (s = 1, 2, 3, 4, \cdots) \end{aligned}\right\}. \quad (44)$$

The number of terms is finite in the Fourier series (43) as well as in the expressions (44).

After these preparations the solution T' of the additional problem can be written in the form

$$\left.\begin{aligned} T' = &\left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{1i}\right]e^{-n_1^2\tau}\cos n_1\zeta \\ &+\left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{3i}\right]e^{-n_3^2\tau}\cos n_3\zeta + \cdots \\ &+\left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{2i}\right]e^{-n_2^2\tau}\sin n_2\zeta \\ &+\left[\sum_{j=0}^{N}\frac{\tau^j}{j!}D^jW_{4i}\right]e^{-n_4^2\tau}\sin n_4\zeta + \cdots \end{aligned}\right\}, \quad (45)$$

wherein N is again a finite number. The verification is left to the interested reader.

NOMOGRAMS FOR THE SOLUTION OF SPHERICAL TRIANGLES.

BY

DR. JAMES B. FRIAUF.

The nomogram described below will give a complete solution of a spherical triangle for which two sides and the included angle or three sides are known, and can also be used, although with very much less convenience, for the solution of a spherical triangle in which the known parts are two sides and the angle opposite one of the known sides. It can be employed, either alone or in conjunction with the other nomograms given here, to make the calculations needed for:

1. The determination of the great circle distance between any two points on the surface of the earth, the initial and final courses, and intermediate points and courses on the great circle track.
2. Navigation by the method of Marcq St. Hilaire.
3. The determination of the declination and hour angle of an observed star for the purpose of star identification.

A number of nomograms have been devised for the solution of spherical triangles. Those which are known to the author are to be found in references 1 to 6 at the end of this paper, and there may well be others. With one exception, the nomograms which are given here are different from any which have been described by other writers on this subject and are believed to have some advantages that will make them of interest.

The nomogram shown in Fig. 1 is basically the same as the one described by the author [6] some years ago for the determination of the great circle distance between two points but has one of the scales graduated differently to facilitate the use of the nomogram for the determination of angles. It covers the entire range of variables that need be considered for the solution of any spherical triangle in which the sides

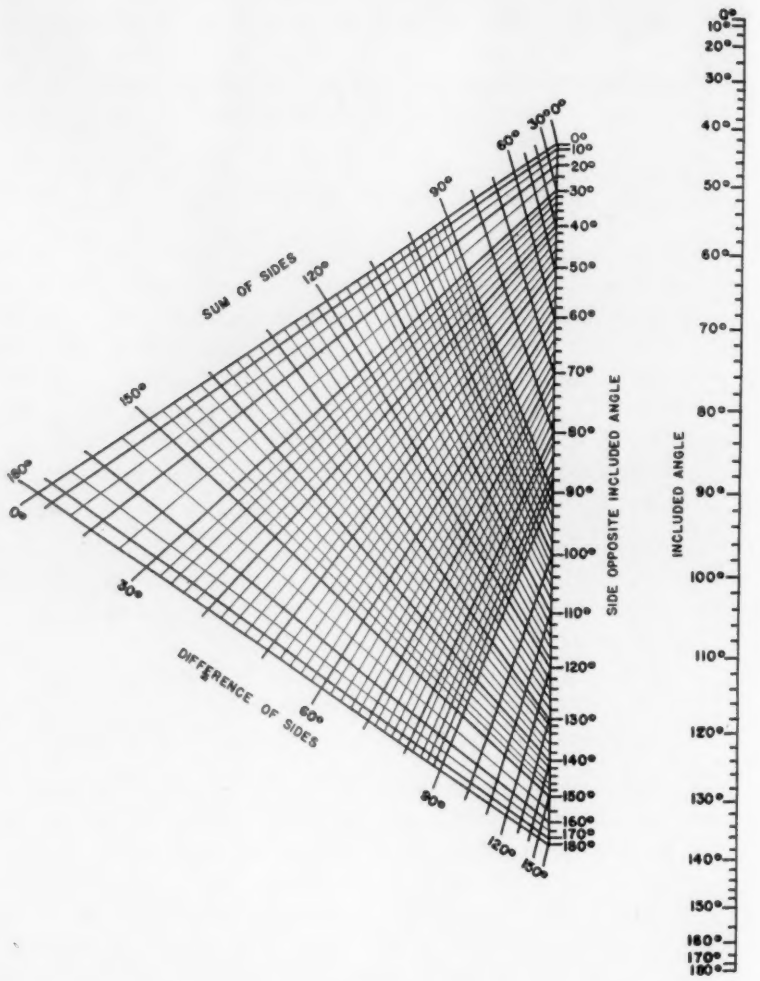


FIG. 1.

and the angles are less than 180°, and has the disadvantages that are not uncommonly associated with nomograms designed to cover a wide range of variables, namely, limited precision unless drawn on a large scale and very poor determination of some of the variables over a portion of their range. These disadvantages can be overcome to a considerable extent by

the construction of auxiliary nomograms which cover only a portion of the range included in the main nomogram. The advantages of the nomographic method of solution are that it can be used quickly and easily without requiring any knowledge of the trigonometrical functions or the formulas of spherical trigonometry; gives the desired angles without any ambiguity as to quadrant; shows directly what effect a change in one of the variables has upon the variable being determined; and even when used with nomograms drawn on a fairly small scale has sufficient precision to be useful for star identification, for sketching great circle courses, and for checking calculations made by more precise methods to guard against the presence of any gross errors.

Consider a spherical triangle with three sides, a, b, and c, and the three opposite angles, A, B, and C. The nomogram in Fig. 1 is based upon the following relation:

$$2\cos c = \cos(a+b) + \cos(a-b) + [\cos(a-b) - \cos(a+b)]\cos C,$$

which can be immediately reduced to the somewhat more familiar relation;

$$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

A similar but not identical relation holds between the three angles and the side opposite one angle. With suitable modifications in its graduations, the nomogram shown in Fig. 1 could also be used for the solution of a spherical triangle in which the known parts are two angles and one side or three angles. For many of the problems of navigation, however, the interest lies in the solution of spherical triangles in which two sides and the included angle or three sides are known, and the accompanying nomogram is so graduated as to apply to this case.

Since there are four variables involved in the equation given above, the nomogram must show four variables. The treatment of an equation with four variables may involve either a nomogram with double alignment, or a nomogram with a network. Neither is as simple or convenient as the nomograms for equations with three variables but the additional complexity is no more than a natural consequence

of the fact that an equation with four variables is more complicated than an equation with only three.

The nomogram in Fig. 1 has a vertical scale graduated to show the included angle, C, a parallel scale for the opposite side, c, and a network formed by two systems of straight lines. The lines of the network which slope upward to the left are graduated to show the sum of the two sides, a and b, which include the angle, C; the lines of the network which slope downward to the left are graduated to show the difference of the same two sides. To use the nomogram to find the side, c, when two sides, a and b, and the included angle, C, are given: (1) locate the line on the network which corresponds to the sum of the given sides; (2) follow this line to its intersection with the line on the network which corresponds to the difference of the same two sides; and (3) pass a straight line from the point of intersection so determined to the included angle, C, shown on the right hand vertical scale. This line cuts the c scale at the value of the side opposite the angle, C. In using a nomogram with a network it will be found convenient to fasten a piece of transparent paper over the nomogram so that the point of intersection of the scales on the network can be marked on the paper and not lost in the subsequent operations.

The range of variables covered by the nomogram is from 0° to 180° for the side, c, the opposite angle, C, and for the sum and difference of the two sides, a and b, which include the angle, C. Since the two sides, a and b, are both less than 180°, their difference will also be less than 180° and can be found immediately on the network. If the sum of a and b is greater than 180°, which may happen, solve the spherical triangle which has the angle, C, included between two sides equal to $180° - a$ and $180° - b$. The difference between the including sides will be the same for the second triangle as for the first, and so too will be the included angle and the side which is opposite it, while the sum for the including sides of the second triangle will be less than 180° instead of more than 180° and can be found on the network.

Three special cases in which the solution of the spherical triangle is obvious are shown very clearly by the nomogram. These are:

1. Sum and difference of a and b equal to 180° and 0° respectively. In this case, both a and b must be equal to 90° and the side, c, opposite the included angle, C, must be equal to the angle. Inspection of the nomogram shows that any index line drawn from the point on the network which corresponds to a sum and difference of 180° and 0°, respectively, cuts the c and C scales at equal values.
2. Included angle, C, equal to 0°. In this case the side opposite the included angle must be equal to the difference between the other two sides and is independent of their sum. This is apparent on the nomogram since the lines of the network which correspond to the difference between a and b all radiate from the upper end of the included angle scale, which corresponds to $C = 0°$, and pass through the points of the c scale which correspond to the difference between a and b.
3. Included angle, C, equal to 180°. In this case the third side is equal to the sum of the other two and is independent of their difference. This condition requires that the lines of the network for the sum of a and b must radiate from the lower end of the C scale.

The nomogram can be constructed graphically of any desired size as follows: Lay off a cosine scale for the included angle, C, to cover the range from 0° to 180°; select a point on the perpendicular bisector of the cosine scale to serve as the point corresponding to a sum and difference of a and b equal to 180° and 0°, respectively; between this point and the C scale draw a line parallel to the C scale to serve as a support for the c scale; project the cosine scale on this line from the chosen point; and then draw straight lines from the ends of the C scale through the graduations of the c scale. These lines will form the network for the sum and difference of a and b. This construction will give a nomogram in which the scales have the same relative positions as in Fig. 1. The point for the sum and difference of a and b equal to 180° and 0° can be taken, however, between the c and C scales, or off to one side of the C scale while the c scale is on the other side. In the first case the network will be between the c and C

scales, instead of outside as in Fig. 1, but would be used in the same way. In the second case, the network would be on the side of the c scale which is opposite the C scale but would run off to an infinite distance instead of terminating at a finite distance.

Since the nomogram is designed to show the relation between the sides of a spherical triangle and the angle included between two of them, it might be thought that it would be more natural to graduate the network directly in terms of the sides, a and b, instead of in terms of their sum and difference. This can be done, and a nomogram has long been known [2] which is constructed in this way. The network so obtained, however, consists of intersecting curves instead of the straight lines that are obtained when the sum and difference of the sides are used. It is less convenient to use, and is far more troublesome to construct. It is believed that the advantages of having a network made up of straight lines instead of curves are more than sufficient to compensate for the additional trouble of having to compute the sum and difference of the sides before entering the network to locate one of the points through which the index line must pass. The nature of the curves formed when the network is graduated directly in terms of the sides instead of their sum and difference can be seen by taking one side equal to a fixed value, 50°, for example, and letting the other side take on successive values of 0°, 5°, 10°, and so on. The sum and difference will be 50° and 50°, 45° and 55°, and 40° and 60°. A curve drawn through these points on the network will correspond to one side equal to 50°.

Three cases can arise when the given parts of a spherical triangle are two sides and an angle or three sides. These cases and the method of using the nomogram to solve them are as follows:

1. *Two sides, a and b, and the included angle, C, are given.* The method of using the nomogram to find the third side has already been explained. When this has been found, all sides are known and the remaining angles can be found by proceeding as for the next case with the nomogram of Fig. 1, or by using the nomogram shown in Fig. 2 and described below.

2. *Three sides, a, b, and c, are given.* Find the point on the network corresponding to the sum and difference of a and b, and pass an index line through this point and the point on the c scale corresponding to side c. This line will cut the C scale at the value of the angle opposite c. Since c can be any side of the triangle, all three angles can be found using only the nomogram of Fig. 1. Alternatively, after one angle has been found in this way, the others can be determined by the nomogram in Fig. 2.
3. *Two sides, b and c, and the angle, C, opposite one side are given.* To solve this case with the nomogram of Fig. 1, pass an index line through c and C, cutting across the network. The lines of the network are graduated in terms of the sum and difference of a and b, that is, in terms of $a + b$ and either $a - b$ or $b - a$ depending upon which is larger. Note mentally the sum and difference of the graduations on the lines of the network which intersect on the index line, and proceed across the network on the index line until a place is found where either the sum or difference of the graduations is equal to $2b$. Since both the sum and difference of the angles, a and b, are known at this point, both a and b can be easily found. The side, b, will, of course, come out to equal to the known side, b; the other is the unknown side which it was desired to determine. Knowing all three sides, the angles can be found as before.

While the nomogram of Fig. 1 can be used for the solution of all three cases given above, the procedure for the third case is highly inconvenient and awkward. This case is very easily handled by the use of the sine formula to determine the third side and the nomogram of Fig. 1 is not recommended. Its utility is much greater when two sides and the included angle, or three sides are given.

The sine formula of spherical trigonometry, which states that the quotient formed by dividing the sine of any angle by the sine of the opposite side is the same for all three pairs of sides and opposite angles in a spherical triangle, can be

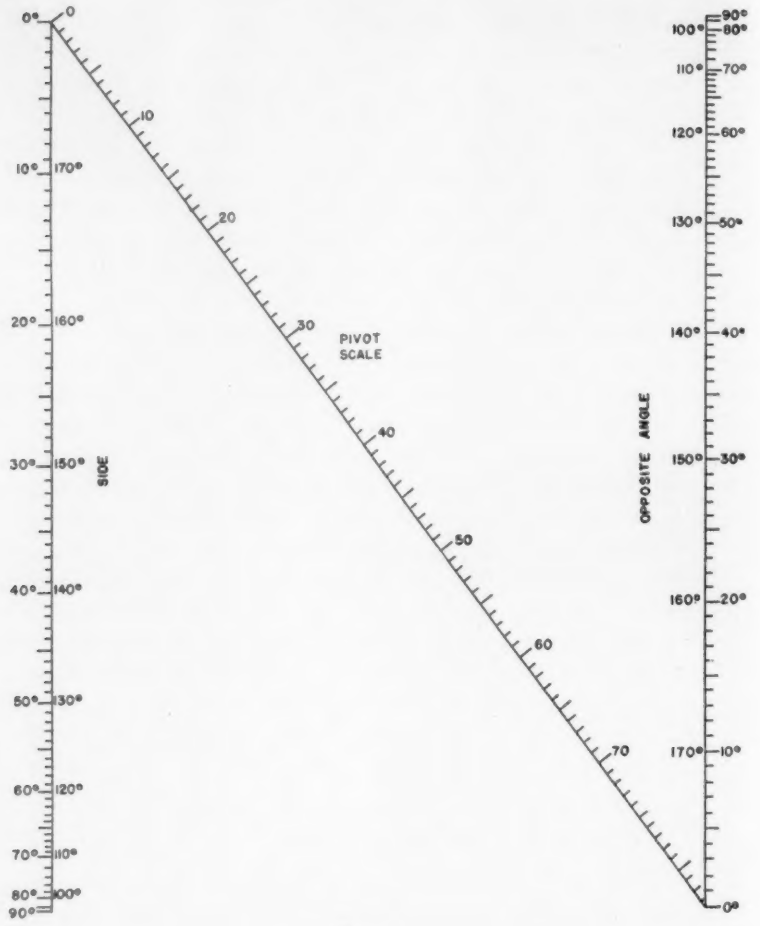


FIG. 2.

handled by means of the well known nomogram [2] shown in Fig. 2. This has a scale graduated to show sides, one to show angles, and a pivot scale. To use this nomogram, one side and its opposite angle must be known, and either another angle or side. Pass an index line from the known side to the opposite angle, and note where it cuts the pivot scale. Rotate the index line about the point of intersection with the pivot scale until the index line passes through either the other known side or angle, as the case may be. The

intersection of the index line with the other scale then gives the opposite angle or side. The graduations on the pivot scale are of no significance and are merely to facilitate identification of the pivot point about which the index line is rotated.

Since the sine of any angle is the same as the sine of its supplement, the nomogram of Fig. 2 furnishes no information as to whether the desired angle or side lies within the first or second quadrant. Any uncertainty can usually be resolved by inspection of the triangle, but if not, by recourse to the nomogram of Fig. 1 which gives the angle without ambiguity. While the nomogram of Fig. 1 can be used to find angles when the three sides are known, it will be seen that the angle is not determined with much precision if the point on the network fixed by two of the sides is close to the scale for the third side, if the index line cuts the C scale at too acute an angle, or if the angle being determined is close to 0° or 180°. In the first two cases, a small error in locating the points through which the index line must pass will cause a considerable error in the angle. In the third case the angle will be determined poorly because the C scale is very crowded towards its extremities. Consequently, it is desirable to use the nomogram of Fig. 2 to obtain a check on the values for angles found from Fig. 1.

These two form a useful complement, one to the other. While the first can be used to obtain a complete solution of a spherical triangle when two sides and one angle or three sides are known, it is very awkward and inconvenient to use when the known parts are two sides and the angle opposite one of them. This is precisely the case which the second nomogram is adapted to handle. Furthermore, the first has an angle scale which is crowded near 0° and 180° but open near 90°, while the second has an angle scale which is crowded near 90° but open near 0° or 180°. An angle that can be determined only poorly from one nomogram can often be found much more satisfactorily from the other. Used together, these nomograms, even if no larger than about seven and a half by ten inches, furnish solutions of a respectable degree of precision except for some ranges of the variables. Means for overcoming this difficulty, at least in part, are considered below.

The following example illustrates the use of the nomograms in Figs. 1 and 2 to solve one kind of problem which arises in navigation.

Example 1. Given a point of departure at Lat. 23° 09′ N., and Long. 82° 21′ W., and a destination at Lat. 53° 27′ N., and Long. 3° 02′ W. Find the great circle distance between these points and the angles between the great circle track and the meridians at the beginning and end.

Solution. The two sides which include the 79° 19′ difference in longitude are 66° 51′ and 36° 33′. The third side of the triangle, the great circle distance, is found from Fig. 1, after which the two angles can be found from either Fig. 1 or Fig. 2. Results:

	Great Circle Distance.	Angle at Start.	Angle at End.
From Fig. 1	65° 30′	40°	84°
From Fig. 2		40°	83°
Calculated	65° 20.0′	40° 05.3′	83° 52.0′

The results for the nomograms were obtained by the use of the original drawings for Figs. 1 and 2 (about $7\frac{1}{2} \times 10$ inches). It will be seen that the nomograms give values which agree well with each other and with the calculated values except for the angle at the end of the great circle track which is determined poorly from the nomogram of Fig. 2 due to the crowded part of the scale in which it occurs. The nomograms would not have to be drawn to an enormously large scale to give results of sufficient precision for estimating great circle distances and the initial and final courses which are readily obtainable from the angles which the great circle track makes with the meridians. After these have been found as outlined above, other points on the great circle track can be obtained by making further use of the same nomograms. The procedure for doing this need not be described, however, since it is much less convenient than the use of another nomogram to be described later.

The following example illustrates the use of Figs. 1 and 2 for making the calculations needed for star identification.

Example 2. In Lat. 22° 15′ N., a star was observed at an altitude of 47° 10′ and azimuth N. 137° E. Find the declination of the star and its hour angle.

Solution. The two sides which include the azimuth are 67° 45′ and 42° 50′. The third side of the triangle, 90° − declination, is obtained from Fig. 1 and the hour angle can then be found from either Fig. 1 or Fig. 2.

	Declination.	Hour Angle.
From Fig. 1	− 11° 0′	28° 0′
From Fig. 2		28° 20′
Calculated	− 10° 31.0′	28° 08.4′

The results obtained are of sufficient precision for use in star identification. If the nomograms were designed to be used exclusively for this purpose the scales could be graduated directly in terms of latitude, declination, azimuth and altitude, instead of in terms of the sides and angles of a spherical triangle. This would make their use somewhat more convenient for this purpose.

It will be seen that the nomogram of Fig. 1 gives a very poor determination of the included side, c, or of the opposite angle, C, when these are small, or close to 180°. This is a consequence of the fact that both the c and C scales are cosine scales which are very crowded for angles near 0° or 180°. A more satisfactory determination for small angles can be obtained by subjecting the nomogram to a projective transformation [7] which opens up the scales. A transformed nomogram is shown in Fig. 3 which is applicable when the difference between the two sides, a and b, which include the angle, C, is between 0° and 28°, while the included angle, C, is between 0° and 32°. This nomogram also gives a poor determination for the third side when it is nearly 0° but can be opened up still more to give the nomogram of Fig. 4. Both these nomograms are used for the same purposes and in the same way as the nomogram in Fig. 1 since they are simply transformations of a portion of this nomogram designed to cover only a small range of variables. The nomograms in Figs. 3 and 4 are primarily for the determination of the unknown side when two sides and the included angle are known. They may also be used, however, to determine an unknown angle but do not cover the entire range of variables necessary to do this in all cases. In such cases the nomograms of Figs. 1 or 2 can be used.

By the use of nomograms such as those shown in Figs. 3 and 4 it is possible to determine the third side of the spherical triangle with considerable precision when the angle opposite the side is small and the difference between the two sides which include the angle is also small. When the included

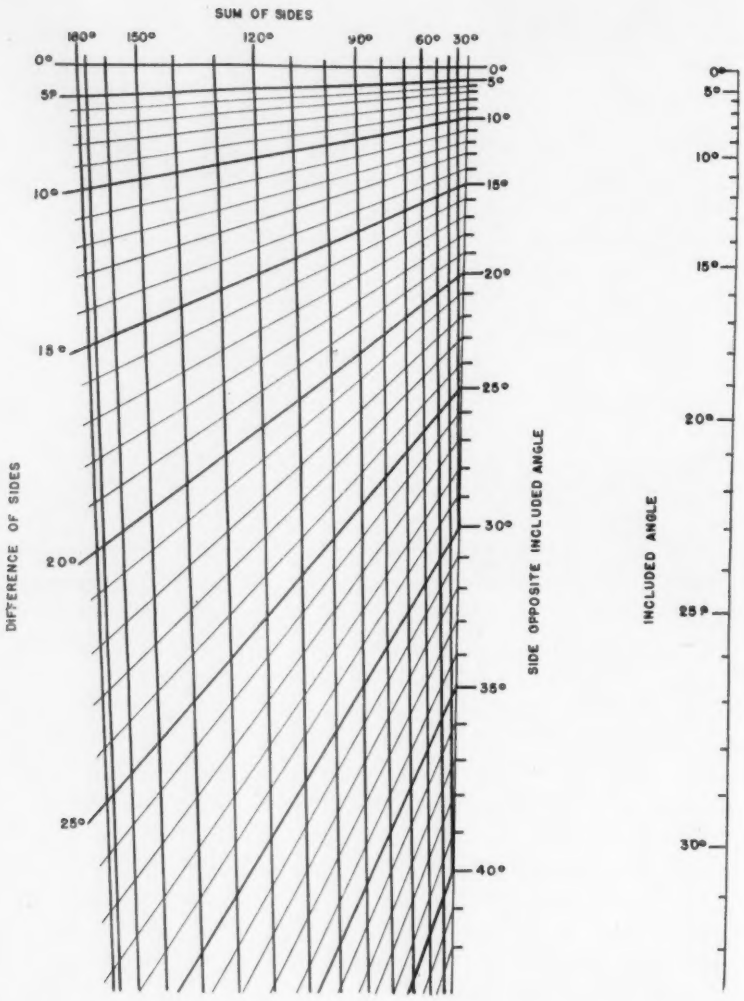


FIG. 3.

angle, C, is small but the difference between the sides which include it is considerable, these nomograms do not apply. In this case, however, the third side, c, is approximately equal to the difference between the other two sides, a and b, and a nomogram has been devised to determine the small correction which must be added to this difference to give the

third side. Let a be the larger of the two sides, a and b, and δ be the correction which must be added to $a - b$ to give the third side, c. Then:

$$\delta = c - (a - b).$$

Also:

$$\begin{aligned} \cos c &= \cos a \cos b + \sin a \sin b \cos C, \\ \cos (a - b) &= \cos a \cos b + \sin a \sin b, \\ \cos c - \cos (a - b) &= \sin a \sin b(\cos C - 1). \end{aligned}$$

By making use of the relations between the different trigonometric functions this relation can be ultimately reduced to:

$$\frac{1}{2} \tan (a - b)\left[\sin \delta + \sin 2a \sin^2 \frac{C}{2} \right] + \left[\sin^2 \frac{\delta}{2} - \sin^2 a \sin^2 \frac{C}{2} \right] = 0.$$

The value of δ which satisfies this equation can be obtained from the rather weird looking nomogram of Fig. 5. To use this, locate the point on the network determined by the intersection of the straight line corresponding to the larger side, a, and the curve which corresponds to the included angle, C. Pass a straight line from this point to the value of $a - b$ as shown on the right hand scale and read off the value of δ where this cuts the correction scale. This value of δ added to $a - b$ gives the third side of the triangle.

Example 3. A spherical triangle has two sides, 110° 27′ and 53° 42′ including an angle of 5°. Find the third side.

Solution. The larger of the two sides is 110° 27′ and the difference is 56° 45′. Results:

	Correction.	Third Side.
From Fig. 5	12.0′	56° 57.0′
Calculated		56° 56.8′

Since the correction is small it can be obtained from the nomogram with considerable precision and gives the third side with no greater error than is made in reading the small correction. Figure 5 covers the range of included angle from 7° to 4°. Below 4° the curves for the included angle come so close together that they are not shown. A projective transformation can be used to expand the part of the network which corresponds to the smaller angles, and at the same time,

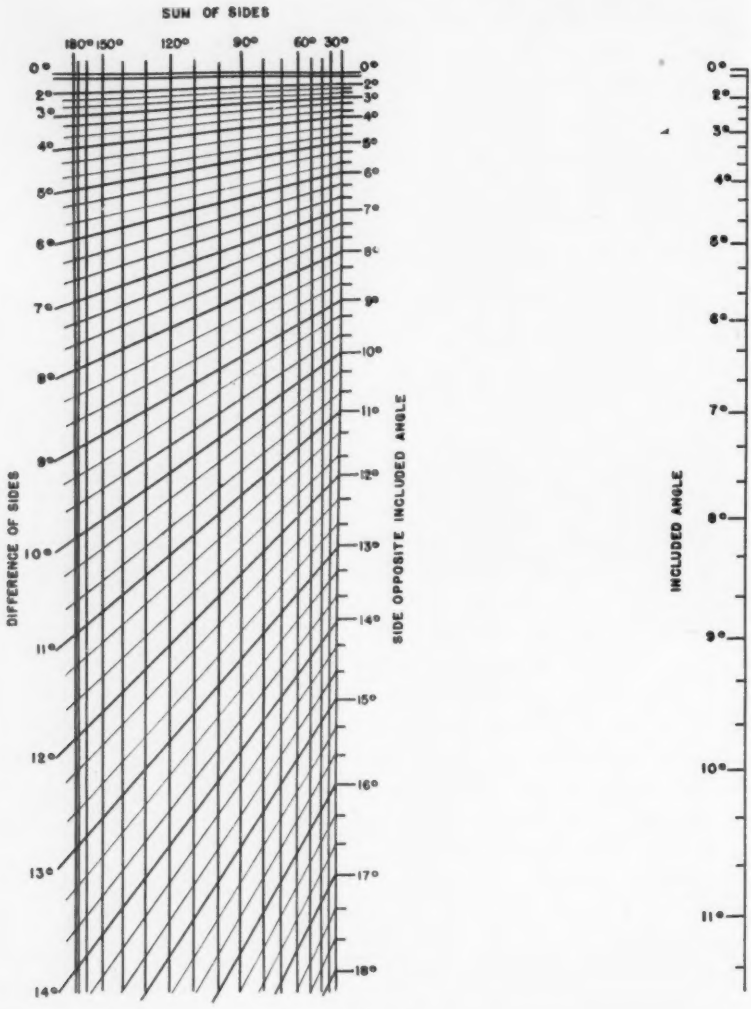


FIG. 4.

if so desired, the nomogram can be made to run to smaller values of $a - b$ than are shown in Fig. 5. Since the relation represented by the nomogram is exact and involves no approximations arising from the special values of the variables represented, it holds as well for values of $a - b$ down to 0° as for those shown. It will be seen, however, that for small

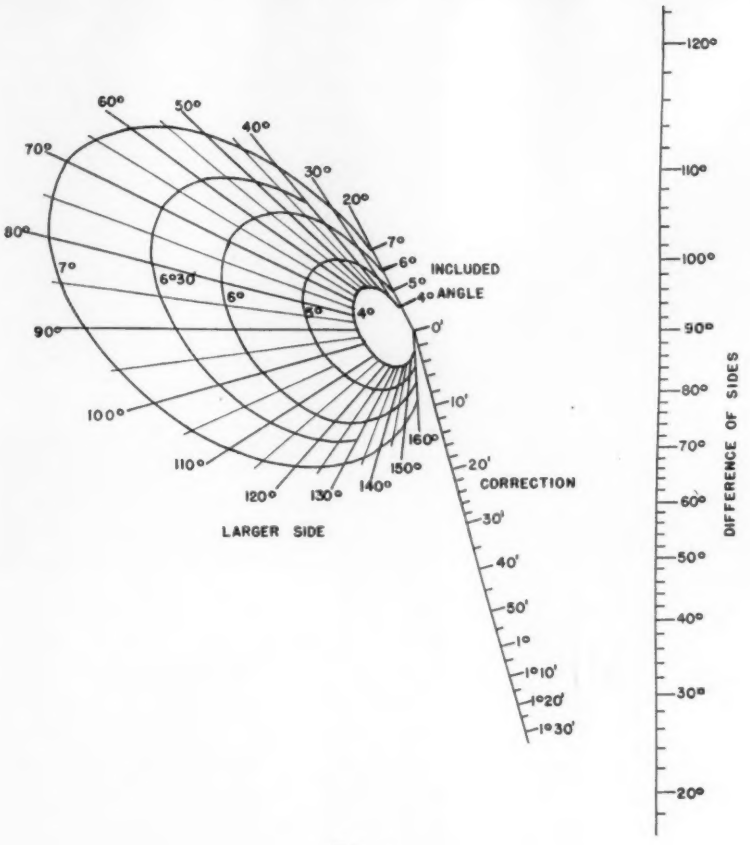


FIG. 5.

values of $a - b$ the index line could cut the correction scale at a very acute angle. This gives a poor determination of the correction, and, as a consequence, no attempt has been made to show values of $a - b$ for less than 18° on the nomogram of Fig. 5. For a transformed nomogram limited to values of C less than 4°, smaller values of $a - b$ could also be shown.

The nomogram shown in Fig. 5 can be obviously used in the Marcq St. Hilaire method of navigation for the determination of the calculated altitude when the hour angle is small. If it is to be used exclusively for this purpose the scales can

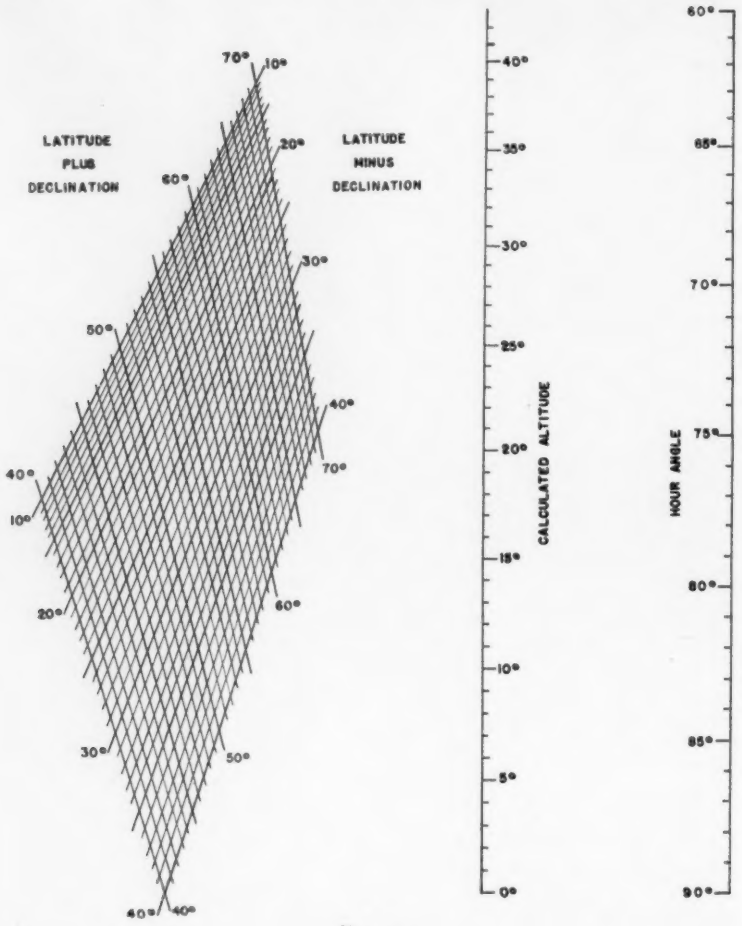


FIG. 6.

be graduated in terms of the latitude of the assumed position, declination of the observed body, hour angle, either in time or in circular measure, and the correction which must be applied to 90° minus the difference between the latitude and declination to give the calculated altitude.

The nomograms shown in Figs. 3 and 4 illustrate the changes that can be effected by a projective transformation to enlarge that portion of the nomogram shown in Fig. 1 which corresponds to small values of C. Other parts of the

nomogram can be expanded equally well to suit special conditions. Consider a plane flying between latitudes 30° and 50° N. and making observations on the sun at a time when its declination is between 10° and 20° N. The maximum range of variables that has to be considered for the sum of the two sides of the astronomical triangle which include the hour angle is from 110° to 140°, and for the difference the range is from 10° to 40°. Only this part of the network is needed and can be enlarged by means of a projective transformation. The nomogram of Fig. 6 covers this range of sides and hour angles of from 60° to 90°. Unlike the nomograms of Figs. 1, 3, and 4, however, its scales are graduated in terms of the latitude of the assumed position, declination of the observed heavenly body, hour angle, and calculated altitude to facilitate its use in making the calculations needed in the Marcq St. Hilaire method of navigation. To use this nomogram, locate the point on the network which corresponds to the sum and difference of the assumed latitude and declination of the observed heavenly body, pass a straight line from this point to the hour angle, and read off the calculated altitude where the index line cuts the calculated altitude scale.

Example 4. At a time when the sun's declination is 15° 27′ N., it is observed from an assumed position in latitude 46° 27′ N., and a longitude which gives an hour angle of 68° 20′. Find the calculated altitude and the azimuth.

Solution. The altitude is determined from Fig. 6, the azimuth from Figs. 1 and 2.

	Calculated Altitude.	Azimuth.
From Fig. 6	26° 01′	
From Fig. 1		94°
From Fig. 2		95°
Calculated	25° 59.5′	94° 45′

The azimuth can be found with sufficient precision for laying off a Sumner line by the use of the nomograms shown in Figs. 1 and 2, while the nomogram of Fig. 6 would not have to be drawn on a very large scale to give altitudes that are close enough to be of real use for aerial navigation. The nomogram of Fig. 6, it will be noted, covers only hour angles from 60° to 90° and would have to be supplemented by others to take care of the smaller hour angles.

The nomogram shown in Fig. 7 can be used for finding the vertex of a great circle track and intermediate points and courses. Let:

L_P be the latitude of any point, P, on the great circle track;
L_V be the latitude of the vertex, V;
B be the difference in longitude between P and V;
A be the angle at P between the direction of the meridian at P which points towards the elevated pole corresponding to a positive sign for L_V, and the direction of the great circle track at P which points towards the vertex, V.

The two latitudes, L_P and L_V, can each range from $-90°$ to $+90°$. It is sufficient, however, to consider only a range of from $0°$ to $90°$ for L_V since the calculations for the determination of the great circle track can always be made with respect to the elevated pole which gives L_V a positive sign. The latitude of P, L_P, will be plus or minus depending upon whether it is on the same side of the equator as the vertex, V, or upon the opposite side. Furthermore, a little consideration will show that since A is measured between the elevated pole which makes L_V positive and the direction of the great circle track which points towards the vertex, A must be less than $90°$. For a given value of L_V, the value of A will range between $90° - L_V$ and $90°$ as B is varied. The difference in longitude, B, between P and V can have any value between $0°$ and $180°$. When B is less than $90°$, L_P is positive, that is, has the same sign as L_V; when B is greater than $90°$, L_P is negative. When B is $90°$, L_P is $0°$ since the great circle track always crosses the equator at $90°$ from the vertex.

The nomogram in Fig. 7 has four scales to show A, B, L_P, and L_V. When any two of these four variables are known, an index line is passed through the points corresponding to the known values. This line cuts the other two scales at the appropriate values for the other two variables. But while the nomogram in Fig. 7 involves four variables just as the nomograms in Figs. 1 to 6 also involve four variables, the kind of relation is quite different. The earlier nomograms give the solution to one equation involving four variables, of which three must be known before the fourth can be found.

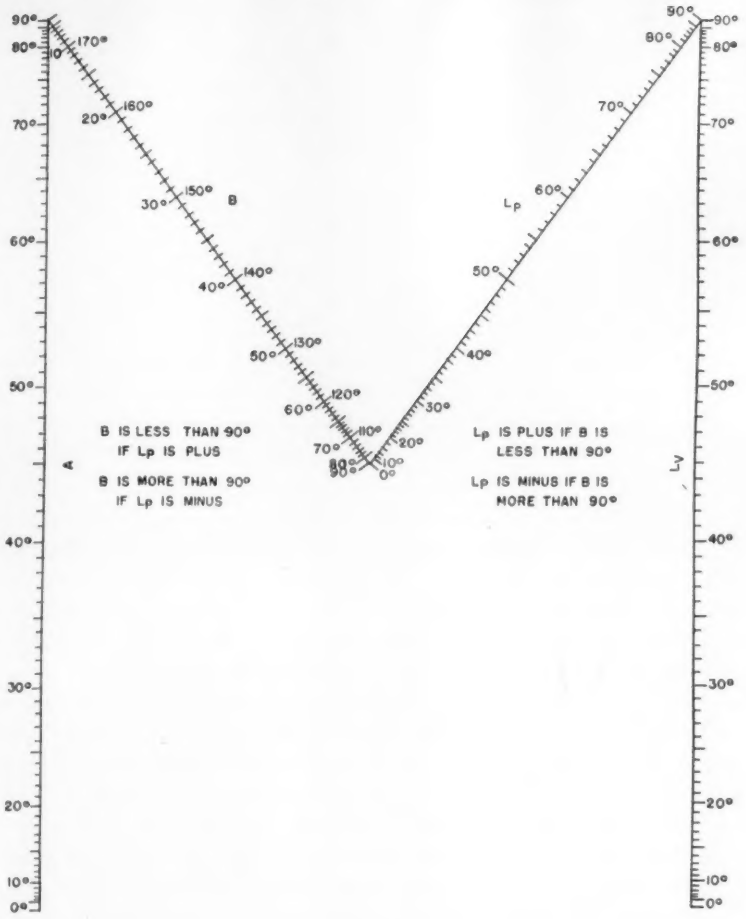


FIG. 7.

The nomogram of Fig. 7 gives the solution to a number of relations between three variables of which two suffice to determine the third. There are four different ways of selecting three variables from the group of four shown in the nomogram of Fig. 7. This nomogram could be split into four separate nomograms, each of which would show the relationship which holds for each of the four groups of three variables, one for A, B, and L_V; one for A, B, and L_P; one

for B, L_P, and L_V; and one for A, L_V, and L_P. It is obviously more useful to have all these relations shown on the same nomogram and Fig. 7 has been arranged to do this.

To use this nomogram in connection with the calculation of great circle courses, first make use of the nomograms in Figs. 1 and 2 to determine the great circle distance and the angles between the great circle track and the meridians at its ends. The angles found from Figs. 1 and 2 are those measured between the meridians and the direction of the great circle track towards the other point. This will also be the direction towards the vertex if the vertex lies between the two points between which the great circle track is drawn. If the vertex lies to one side of the two points, one of the angles found from Figs. 1 or 2 will be less than 90° and the other will be greater than 90°. The one less than 90° will be the angle between the meridian and the direction of the track towards the vertex; the one greater than 90° will be the angle between the meridian and the direction of the track away from the vertex. This should be subtracted from 180° to give the angle, A, as used in the nomogram of Fig. 7 which is always measured towards the vertex, and is always less than 90°.

Having found these angles by the use of the nomograms in Figs. 1 and 2, locate the latitude of the initial point of the great circle track on the L_P scale of Fig. 7 and pass an index line to the value of A found for the initial point. This index line will cut the other scales at the latitude of the vertex and the difference in longitude from the vertex to the initial point. Repeat this procedure using the latitude of the final point and the value of A found for this point. The same value for the latitude of the vertex should obviously be obtained, but except for exceptional cases the longitude from the vertex to the final point will be different than to the initial point. A check on its correctness can be obtained from the consideration that the longitude of the vertex should come out to be the same whether it is determined from one end of the great circle track or the other. Additional points on the great circle track corresponding to differences in longitude of 5°, 10°, 15°, or any desired number of degrees from the vertex can be found by passing an index line from

L_V to the appropriate value of B and reading off the corresponding values of L_P and of the angle, A, from which the course at the intermediate point is readily obtained.

Example 5. Locate the vertex for the great circle track of Example 1 and find the latitude, L_P, and the angle, A, for points at 20° intervals of longitude from the vertex.

	From Nomogram of Fig. 7.		Calculated.
	Using Initial Point.	Using Final Point.	
Latitude of vertex	53° 45′	53° 45′	
Longitude of vertex	10° 21′	11° 02′	
Mean latitude	53° 45′		53° 41.6′
Mean longitude	10° 41′		10° 39.6′

	Intermediate Points.	
	Nomogram.	Calculated.
(1) *Longitude 30° 41′*		
Latitude, L_P	52° 00′	51° 58.5′
Angle, A	74° 00′	73° 59.0′
(2) *Longitude 50° 41′*		
Latitude, L_P	46° 06′	46° 11.2′
Angle, A	58° 50′	58° 47.1′
(3) *Longitude 70° 41′*		
Latitude, L_P	34° 00′	34° 13.1′
Angle, A	45° 30′	45° 43.6′

It is apparent from inspection of the nomogram that it will give a poor determination of L_P when B is nearly 90° and of A when B is nearly 0°. These deficiencies can be remedied by the use of projective transformations to suitably enlarge the appropriate parts of the nomogram. The transformed nomograms would each be for only three variables: one for L_V from 0° to 90°, B for a small range near 0°, and A for a small range near 90°; and the other for L_V from 0° to 90°, B for a small range near 90°, and L_P for a small range near 0°. With these auxiliary nomograms to take care of the latitude near the equator and the angle A, which determines the course, near the vertex, it is believed that the nomographic solutions would be of sufficient accuracy to be of real utility in laying out great circle courses.

Since the principles of nomography have been treated by a number of authors, one very excellent discussion being found in the book of Hewes and Seward referred to above, it will be unnecessary to enter upon any extended discussion of the theory underlying these nomograms but will be suffi-

cient to give the equations of the scales for those which are shown here. These are:

For Fig. 1

C scale $\quad x = 4,$

$$y = 10 \cos C,$$

c scale $\quad x = 0,$

$$y = \frac{22 \cos c}{3},$$

Network

$$x = \frac{88[\cos (a + b) - \cos (a - b)]}{60 + 22[\cos (a + b) - \cos (a - b)]},$$

$$y = \frac{220[\cos (a + b) + \cos (a - b)]}{60 + 22[\cos (a + b) - \cos (a - b)]}.$$

For Fig. 2

Side scale $\quad x = 0,$

$$y = 20(1 - \sin a),$$

Angle scale $\quad x = 15,$

$$y = 20 \sin A,$$

Pivot scale Graduations arbitrary since they do not enter into the final result but the pivot scale must run as indicated, that is, from the zero of one of the scales to the zero of the other.

For Fig. 3

C scale $\quad x = 6,$

$$y = -125(1 - \cos C),$$

c scale $\quad x = 0,$

$$y = \frac{-1000(1 - \cos c)}{13.5},$$

Network

$$x = \frac{48[\cos (a + b) - \cos (a - b)]}{27 + 8[\cos (a + b) - \cos (a - b)]},$$

$$y = \frac{-2000 + 2000 \cos (a - b)}{27 + 8[\cos (a + b) - \cos (a - b)]}.$$

For Fig. 4

C scale $\quad x = 9,$

$$y = -1000(1 - \cos C),$$

c scale $\quad x = 0,$

$$y = -400(1 - \cos c),$$

Network
$$x = \frac{9[\cos(a+b) - \cos(a-b)]}{5 + \cos(a+b) - \cos(a-b)},$$
$$y = \frac{-2000 + 2000\cos(a-b)}{5 + \cos(a+b) - \cos(a-b)}.$$

For Fig. 5

$a - b$ scale
$$x = 5,$$
$$y = \frac{-7.4}{0.44 + \tan(a-b)},$$

δ scale
$$x = \frac{1.1\sin\delta - 5\sin^2\frac{\delta}{2}}{0.0065 + 0.22\sin\delta - \sin^2\frac{\delta}{2}},$$
$$y = \frac{-3.7\sin\delta}{0.0065 + 0.22\sin\delta - \sin^2\frac{\delta}{2}},$$

Network
$$x = \frac{-(1.1\sin 2a + 5\sin^2 a)\sin^2\frac{C}{2}}{0.0065 - (0.22\sin 2a + \sin^2 a)\sin^2\frac{C}{2}},$$
$$y = \frac{3.7\sin 2a\sin^2\frac{C}{2}}{0.0065 - (0.22\sin 2a + \sin^2 a)\sin^2\frac{C}{2}}.$$

For Fig. 6

Hour angle scale, H
$$x = \frac{125}{22},$$
$$y = 40\cos H,$$

Altitude scale, h
$$x = 0,$$
$$y = \frac{88}{3}\sin h,$$

Network for latitude, L and declination, d
$$x = \frac{-125[\cos(L+d) + \cos(L-d)]}{60 - 22[\cos(L+d) + \cos(L-d)]},$$
$$y = \frac{880[\cos(L-d) - \cos(L+d)]}{60 - 22[\cos(L+d) + \cos(L-d)]}.$$

For Fig. 7

A scale $\quad x = 0,$

$$y = 20 \sin^2 A,$$

L_V scale $\quad x = 15,$

$$y = 20 \sin^2 L_V,$$

B scale

$$x = \frac{15 \sin^2 B}{1 + \sin^2 B},$$

$$y = \frac{20}{1 + \sin^2 B},$$

L_P scale

$$x = \frac{15}{1 + \cos^2 L_P},$$

$$y = \frac{20}{1 + \cos^2 L_P}.$$

June, 1939

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

SULFATE RESISTANCE OF PORTLAND CEMENTS.

In extensive areas of the West-Central portion of the United States and Canada, many clays and soils occur which contain several per cent. of sulfates, particularly sodium sulfate. The action of these "alkali" soils on portland cement mortar or concrete results in the conversion of the hydrated calcium aluminate of the set cement to calcium sulfoaluminate and gypsum, accompanied by marked expansion. Complete disintegration of the mortar or concrete may eventually occur.

Methods by which the sulfate resistance of portland cement mortar or concrete may be improved, include the following: (1) Steaming the mortar or concrete; (2) prolonged curing of the mortar or concrete at ordinary temperatures; (3) mixing the cement with a puzzolanic material; (4) increasing the glass content of the cement; (5) altering the chemical composition of the cement.

Experiments by E. P. Flint and Lansing S. Wells which are reported in the *Journal of Research* for August (RP 1411), show that the above methods are effective because they bring about the removal from the set cement of tricalcium aluminate hexahydrate, which is unstable in sodium sulfate solutions, and the formation in its place of silica- or iron-containing hydrogarnets, which are stable in such solutions.

Products having an X-ray structure nearly identical with that of grossularite garnet were obtained when mixtures of beta-dicalcium silicate, lime, and dehydrated kaolin were steamed at 500° C. and 420 atmospheres. Dicalcium silicate and tricalcium silicate are major constituents of portland cement and dehydrated clay, or kaolin, is a common puzzolanic cement admixture.

* Communicated by the Director.

BEHAVIOR OF CALCIUM SULFATE AT HIGH TEMPERATURES.

The behavior of pure calcium sulfate is of interest in considering the properties of those types of calcium sulfate cements or plasters which are prepared by heating the raw material to a very high temperature. Several plants are in existence abroad at which calcium sulfate is used as one of the raw materials for the manufacture of portland cement and sulfuric acid, advantage being taken of the decomposition of calcium sulfate to lime and sulphur trioxide at high temperatures. This decomposition is presumably responsible in part for the absence of $CaSO_4$ in many portland cement clinkers made in the ordinary way, as has been reported recently by this Bureau.

Naturally occurring calcium sulfate is found in two forms, as gypsum with two molecules of water of crystallization and as anhydrite with no water of crystallization. The latter form is referred to as the beta, or ordinary low temperature modification. Two other forms can readily be prepared, the hemihydrate, $CaSO_4.1/2H_2O$, of which plaster of Paris is largely composed, and an extremely hygroscopic anhydrous form, soluble anhydrite, which changes rapidly to hemihydrate at ordinary temperatures in the presence of water vapor. A third form of anhydrous calcium sulfate occurs but is formed and exists only at high temperatures. This is known as the alpha modification and changes rapidly to the ordinary form when cooled.

In an investigation by Edwin S. Newman reported in the August *Journal of Research* (RP 1413), differential heating curves were used to study the behavior of calcium sulfate at high temperatures. After cooling, the material was examined by means of the petrographic microscope and by study of its X-ray pattern. It was found that the beta- to alpha-transition occurs at approximately 1214° C. and the conclusion of its discoverer—that the high temperature form cannot be cooled to room temperature for study—was confirmed. At the high temperatures at which the alpha modification is formed, a eutectic mixture of alpha calcium sulphate and the lime formed by the decomposition of $CaSO_4$ appears to exist. The eutectic temperature is approximately 1365° C. The existence of basic sulfates of calcium which have been

reported by other writers was not confirmed by this investigation.

RESISTANCE OF PLASTICS TO CHEMICAL REAGENTS.

In tabulating data on plastics for handbooks and manufacturers' bulletins, information on the effect of chemical reagents is generally included. The selection of compounds and concentrations for use in such tests was given early consideration by ASTM Committee D-20 on Plastics, and Tentative Method of Test D543–39 T for resistance of plastics to chemical reagents was included in the first group of methods adopted by that Committee. This not only provided a uniform test procedure for use in purchase specifications, but insured comparable data on new and on the older varieties of plastics.

The Organic Plastics Section of the Bureau cooperated in the exploratory investigation undertaken while preparing the ASTM test method. The following plastics were examined: Molded, cast and paper-base laminated phenol-formaldehyde resin; molded and paper-base laminated urea-formaldehyde resin; molded and cast polystyrene; cast methyl methacrylate resin; vinyl chloride-acetate resin; vinyl butyral resin; cold-molded bituminous plastic; cold-molded phenolic plastic; cellulose nitrate; cellulose acetate; ethylcellulose; and casein plastic. The resistance of these materials to all standard and supplementary reagents listed in the ASTM method was determined. These included weak and strong acids, weak and strong alkalies, salt solutions, hydrogen peroxide, and organic solvents. Changes in the weight, dimensions, and appearance of the test specimens were recorded. These experimental data were reported in a paper by G. M. Kline, R. C. Rinker, and H. F. Meindl, presented at the annual meeting of the American Society for Testing Materials in Chicago on June 24.

The limitations of these results as well as those of other tests for determining the permanence of plastics should be recognized. The choice of types and concentrations of reagents, duration of immersion (7 days), temperature of the test (25° C.), and properties to be reported upon, is necessarily

arbitrary and serves primarily as a guide to investigators who wish to compare the relative resistance of various plastics to chemicals. For applications involving continuous immersion, the data obtained in a short-time test are of interest only in eliminating the most unsuitable materials. Those with special problems must necessarily modify the duration of contact with the chemical, the temperature of the system, and such physical tests as may be used, to meet their particular needs.

CHEMICAL DURABILITY OF GLASS.

Donald Hubbard and Edgar H. Hamilton have applied an interferometer method for determining the chemical durability of glass to a series of glasses over a wide range of conditions of time, temperature, and pH. The results, which are given in the August *Journal of Research*, emphasize the fact that there can be no single test by which the durability of glass can be judged with respect to all conditions of service. The method has been particularly helpful in accounting for many of the anomalies of the glass electrode.

CHEMICAL REACTIONS OF THE CHLORITES WITH CARBOHYDRATES.

Interest in the chemistry of the chlorites has recently been stimulated by the commercial production of sodium chlorite. Until a few months ago the literature afforded only meager information about the general chemistry of the chlorites, and knowledge of their chemical behavior with the carbohydrates was limited to the fact that sodium chlorite bleaches the objectionable colored substances in paper and cotton textile materials without weakening the cellulose fibers. A reagent possessing the desirable properties of sodium chlorite should find extensive use in the field of carbohydrate chemistry. To provide a guide for the application of the reagent and to determine how the characteristic groups and structures occurring in the carbohydrates react with chlorites, Allene Jeanes and Horace S. Isbell undertook a systematic investigation of the reactions of chlorites with carbohydrate material of diverse character.

As set forth in the August number of the *Journal of Research* (RP 1408), this investigation has shown that at room temperature the nonreducing disaccharide, sucrose, is completely inert to chlorites under all conditions which do not involve acid hydrolysis, and that the ketoses, the polyhydroxy alcohols, and the aldonic acids are attacked only after treatment with chlorites for many days. In marked contrast to the glycosides, polyhydroxy alcohols, aldonic acids, and ketoses, the aldoses are oxidized readily and give the corresponding aldonic acids.

The aldopentoses are oxidized more rapidly than the aldohexoses and the monosaccharides more rapidly than the disaccharides. The reaction is slow in neutral solution but rapid in acid solution. Quantitative measurements under various conditions have revealed that chlorous acid is the oxidant and that the reaction corresponds in large measure to the following equation:

$$(\text{Aldose}) + 3HClO_2 \rightarrow (\text{Aldonic Acid}) + 2ClO_2 + HCl + H_2O.$$

THERMAL EXPANSION OF CHROMIUM.

In the *Journal of Research* for August (RP 1407), Peter Hidnert presents data on the linear thermal expansion of six samples of chromium (99.2 to 96.3 per cent.) at various temperatures between − 190° and + 707° C. The expansion curves indicated anomalies at low temperatures. These anomalies were reversible on heating and cooling. The temperature at which anomalous expansion occurs, appears to decrease with decrease in the purity of the chromium. Hysteresis was not observed on heating cast or swaged chromium to 300° C. and cooling to low temperatures. Exceptionally low average coefficients of expansion were found for temperature ranges in or near the anomalous critical regions. The average coefficients of expansion of the samples of chromium from 20° C. to various temperatures between 100° and 700° C., lie between 5.7×10^{-6} and 10.3×10^{-6} per degree C.

TOUGHNESS OF MEDIUM-CARBON FORGING STEEL.

In order to guard against sudden failure of materials used in structures, engineers demand a certain amount of toughness in steels. A common method of evaluating this quality is the impact test of notched specimens, and factors affecting the impact-toughness of steels are naturally of considerable interest to engineers.

Samuel J. Rosenberg and Daniel H. Gagon have studied the relation between grain size and heat treatment and the impact-toughness of steel, particularly as affected by low temperatures. Most steels lose toughness (i.e., become brittle) quite rapidly as the temperature is decreased. The range of temperature wherein this change occurs is a reliable criterion of the relative toughness of steels—the lower the temperature at which toughness is lost, the better the steel.

As explained in RP 1410 in the August *Journal of Research*, six heats of the same type of steel (a medium-carbon forging steel), made so as to have different grain sizes, were studied. It was found that, as hot rolled, no relation existed between grain size and toughness, and all the steels were quite brittle. Normalizing (heating to 1600° F. and air cooling) caused some improvement in toughness, and when so treated the fine-grained steels were tougher than the coarse-grained steels. Proper heat treatment (hardening and tempering) resulted in a marked improvement in the toughness of the steels, but in this condition there was no relation between grain size and toughness in the heat treated steels.

The authors conclude that each individual heat of this type of steel has an inherent resistance to impact, characteristic of that particular heat, and that this impact resistance is dependent upon factors not at present recognized.

CODE FOR ELECTRICITY METERS.

The fourth edition of the Code for Electricity Meters was approved by the American Standards Association on May 5, 1941, and was issued in June. The scope of the Code as defined by the ASA is as follows:

Standards of practice for the maintenance and accuracy of watthour meters, demand devices and auxiliary apparatus. Summary of good practice for the installing of meters and auxiliaries. Definitions of units and technical terms relating to watthour meters.

The present edition was prepared by an ASA sectional committee including representatives of 6 organizations in the electrical industry, 3 State utility commissions, and the National Bureau of Standards, under the joint sponsorship of the Bureau, the Association of Edison Illuminating Companies, and the Edison Electric Institute. Copies of the Code (128 pages, bound in cloth) may be obtained for $2.00 each from the Edison Electric Institute, 420 Lexington Avenue, New York, or the American Standards Association, 29 West 39th Street, New York.

Report of the National Advisory Committee for Aeronautics.—The work of this Committee, which has been in existence for twenty-six years, is well known in aeronautical circles and to the public generally through published reports and accounts of its activities. Because of world conditions today the Committee has found it necessary in the national interests to withhold from public distribution the detailed results of its researches. Therefore it is stated, this and succeeding Annual Reports will deal only in general terms with the results accomplished, until world conditions change. Nevertheless, there is much of interest in the report. It is revealed that the outstanding trend in aircraft development produced by the present way in Europe is toward aircraft having higher speeds and greatly increased armor and armament. The demand for increased speed has resulted in the need for much greater horsepower. Whereas pursuit airplanes of a year ago were equipped with engines of 1000 horsepower, they are now being designed with single engines of 2000 horsepower. The trends toward increased speed and higher ceiling, toward larger and heavier engines, toward increased armor and armament, necessitate larger and much heavier types of airplanes. This condition has established a definite trend toward higher wing loadings. Because of the higher flying speeds demanded by the Army and Navy, the Committee has given special study to the important subject of compressibility shock encountered at high speeds. With speeds now attainable, it is essential that care be taken to design all parts of the airplane structure so as to prevent velocities approaching the speed of sound from occurring at any point. The 500 mile-per-hour wind tunnel at the Committee's Langley Field Laboratory has proved of great value in the study of this problem. With increased wing loading, it has become necessary to extend the study of improved high lift devices and of lateral-control devices other than ailerons so as to permit the development of a full-span flap. The trend toward higher speeds has brought many attendant problems, such as the need for improved methods of cowling and cooling of engines, the design of wing ducts, and of ducts for the cooling of engine auxiliaries. High operating speeds also make it necessary to give special attention to the design of air scoops and ventilators.

R. H. O.

THE FRANKLIN INSTITUTE.

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NOTES FROM THE BARTOL RESEARCH FOUNDATION.

THE RELATIVE STOPPING POWERS OF CARBON AND LEAD FOR SLOW MESONS.*

BY

MARTIN A. POMERANTZ AND THOMAS H. JOHNSON

An experimental determination of the relative stopping powers of carbon and lead for slow mesons has been made with an anticoincidence arrangement of G-M counters. For mesons having an average energy of approximately 4×10^7 ev., the experiments revealed that 28.5 g./cm². of carbon is equivalent in stopping power to 24 ± 5 g./cm². of lead. This gives a value for the ratio of the stopping powers of equal masses of carbon and lead, $S_C/S_{Pb} = 0.84 \pm 0.18$, to be compared with the theoretical value of $S_C/S_{Pb} = 1.82$ calculated with the ionization theory. However, it is not necessary to invoke a hitherto unknown absorption process to account for this discrepancy, since it may be explained as arising from the scattering and transition effects. The data corrected for these effects show that 28.5 g./cm². of carbon is equivalent in stopping power to 45 ± 7 g./cm². of lead, and $S_C/S_{Pb} = 1.6 \pm 0.3$, in agreement with the theoretical value within the experimental uncertainty. These experiments indicate that in a dense absorber, any additional absorption process is unimportant compared with ionization for mesons having energies of about 4×10^7 ev., or higher.

THE EAST-WEST ASYMMETRY OF THE COSMIC RADIATION IN HIGH LATITUDES AND THE EXCESS OF POSITIVE MESOTRONS.†

BY

THOMAS H. JOHNSON.

The slight east-west asymmetry of the cosmic radiation in high latitudes, now confirmed by Seidl, is interpreted to be

* An abstract of a paper published in full in *The Physical Review*. **59**, 143 (1941).

† An abstract of a paper published in full in *The Physical Review*, **59**, 11 (1941).

the result of the deflection by the earth's magnetic field of the mesotron component while the rays are losing energy by ionization in the atmosphere. Since this component contains about twenty per cent. more positive than negative rays these deflections result in an asymmetry. Orbits of rays, including those in the range of energy where rest mass cannot be neglected, have been investigated and the deflections determined. It is assumed that deflections without energy loss, namely, those of the primary rays described by the theory of Lemaître and Vallarta, result in a symmetrical distribution for the energy ranges concerned. The asymmetry is traced to the *difference* between the actual deflection and that of a ray which loses no energy. The "difference" deflection δ shifts the angular distribution so that rays which, in the absence of a field, would have produced an intensity proportional to $\cos^2 \zeta$ at zenith angle ζ actually produce this intensity at an angle $\zeta + \delta$. Asymmetries calculated in this way agree with the observed values, and give the correct variation of asymmetry with zenith angle and elevation. Although the data are meager, the theory seems to be in accord with the existing evidence regarding the effect of absorbing material upon the asymmetry.

A REGULATED BATTERY-OPERATED HIGH VOLTAGE SUPPLY.*

BY

J. GRIFFITHS BARRY.†

A portable high voltage supply suitable for Geiger-Mueller counter use has been described by Huntoon,[1] the essential elements being a form of induction coil, a rectifier, and a condenser for smoothing the rectified pulses from the induction coil. Replacing the usual mechanical vibrator type of interrupter is a vacuum tube which has its grid periodically driven beyond cut-off by an auxiliary circuit, thus breaking the flow of current through an iron-cored inductance in the plate circuit.

For use with a radio-balloon cosmic-ray outfit, it was

* Condensed from an article published in full in the *Review of Scientific Instruments*, **12**, 136 (1941).

† Now at the School of Engineering, Princeton University.

[1] R. D. Huntoon, *Rev. Sci. Inst.*, **10**, 176–178 (1939).

deemed advisable to add some form of stabilizer to the circuit in order to minimize voltage variations due both to exhaustion of necessarily small supply batteries and to changes in load current produced by variations of leakage current or by changes in the counting rate of Neher-Harper controlled counters. To minimize power consumption, stabilization was obtained by controlling the input to the voltage-transforming part of the circuit, as shown in the diagram of Fig. 1. By

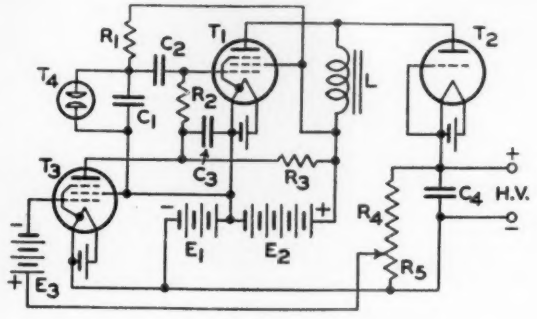

FIG. 1.

R_1 = 0.5–1.0 megohms
R_2 = 0.1 megohms
R_3 = 2.0 megohms
R_4 = 100 megohms
R_5 = 1.5 megohms (Approx. for 1500 volts)
C_1 = 0.006 mf.
C_2 = 0.001 mf.
C_3 = 0.1 mf.
C_4 = 0.1 mf. (1600-volt vibrator condenser)

E_1 = 22.5 volts
E_2 = 135 volts
E_3 = 22.5 volts
T_1 = Type 1B4 tube
T_2 = Type '30 tube
T_3 = Type 1B4 tube
T_4 = ¼-watt neon bulb with resistance removed
L = Audio-transformer (see text)

means of the voltage divider $R_4 - R_5$ and the bias battery E_3, variations in the output voltage are impressed on the grid of the regulator tube T_3, the plate of which controls the steady bias of the interrupter tube T_1, and hence also controls the maximum current through L. For convenience in adjusting the high voltage, the battery E_3 may be connected to the movable arm of a potentiometer (0.1 megohm or more) placed in the voltage divider between R_4 and R_5. Battery E_1 furnishes negative voltage for biasing T_1 and screen voltage for T_3, while most of the input power, exclusive of filament consumption, is supplied by the plate battery E_2. Battery E_3 is for bias purposes only and supplies no power. When used with Geiger-Mueller counter circuits requiring the negative side of the high voltage to be grounded, the batteries E_1 and E_2 may be used as bias and plate supply for the amplifier circuits.

The inductance L is one of the most important elements of the voltage supply. In the final circuit, a type A-16 midget audio-transformer made by the United Transformer Co. was used with both windings in series, and, while rather expensive, was superior to others tested.

Performance tests of a unit operating at 1500 volts showed a decrease of about 1 volt per 10 microamperes output current, the current drain on E_2 being 5 milliamperes at no-load and 9.5 milliamperes at 80 microamperes output. A 10 per cent. change in filament voltage of T_3, potential of E_2, and potential of E_1 produced respective changes of 4.4, 6.7, and 17.7 volts in the output potential at no-load, the figures being only slightly different for 100 microamperes output current. Large changes in temperature changed the output voltage through changes in condenser leakage and also through changes in the values of the carbon resistors used.

The voltage supply described can be made quite compact and can be constructed for a total cost for parts, exclusive of batteries, of less than $10. Without special efforts to reduce weight, as by debasing tubes, the unit weighs about 475 grams. This includes one-half of a Burgess V30FLN battery for E_3, but no other batteries. To prevent corona difficulties at low pressures, a unit was completely inclosed in a metal can about $3\frac{1}{2}''$ in diameter and $7\frac{1}{2}''$ long, with provisions for making the necessary adjustments before final sealing. The battery E_3 and the filament battery of T_2 were placed in the can, the weight of which was then 830 grams. This type of construction seems to be the most practical for balloon use, as it is least affected by external conditions.

In addition to its use in cosmic-ray balloon outfits, the regulated high voltage supply described above is well suited for use with any kind of portable, battery-operated equipment requiring a source of high voltage capable of furnishing a hundred microamperes or more at voltages up to about 2000 volts.

The author wishes to acknowledge the suggestions given by Dr. T. H. Johnson of the Bartol Foundation and also the financial assistance received from the Carnegie Institution of Washington.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Seminar on Polysaccharides and Proteins in the Vibrios.—On May 8, 1941, DR. RICHARD W. LINTON, from the Department of Pathology, Cornell University Medical College, New York—formerly Officer-in-Charge, Cholera Inquiry, Indian Research Fund Association, Calcutta, India—(now of the Biochemical Research Foundation Staff) addressed a seminar group of the Biochemical Research Foundation on the subject of vibrios.

Some years ago Dr. Linton was given the opportunity of going to India to work on cholera. He chose to study the chemical structure of the vibrios, in order to see whether any conclusion could be reached as to which strains of vibrios were dangerous and which harmless. At that time it was difficult, if not impossible, to ascertain serologically whether a given vibrio, unless isolated directly from a clinical case of the disease, would be pathogenic. The situation was further complicated by the fact that in cholera there is no animal test for pathogenicity that can be considered reliable.

The first work concerned the structure of the polysaccharides. During the investigation of a large number of strains from diverse sources Dr. Linton was able to isolate and identify three of these polysaccharides, each organism or strain appearing to have one of them and one only. The first type of polysaccharide consisted of an easily hydrolyzable portion of galactose together with an aldobionic acid made up of galactose and glycuronic acid; the second type had a similar aldobionic acid, and the easily hydrolyzable portion in this instance was arabinose; in the hydrolysis of the third type, glucose was the only reducing substance which could be identified, although the curve of hydrolysis rate against time was not smooth, indicating that the structure was not as simple as would be obtained if a mere polymer of glucose were present. No aldobionic acid occurred in this third type.

These polysaccharides were present in the acetylated form in the living bacterial cell; they differed in specific rotation

and all contained about 3 per cent. total nitrogen and 0.5 per cent. of amino nitrogen. They were, accordingly, similar in structure to polysaccharides which have been found in other bacteria, e.g. *Pneumococcus*, *Shigella dysenteriae*, and Friedländer's bacterium.

In studying the proteins of the vibrios, it was decided to keep the manipulations at a minimum, in order to see if a simple differentiation might be possible. This attempt was successful, since it was found that the so-called "racemization" method gave but two proteins from the whole group, and again as in the case of the polysaccharides, each strain appeared to have but one of these proteins. The method consisted in taking up the protein in *N*/2 NaOH and reading the specific rotations at intervals, frequently within the first few hours, and at daily intervals thereafter. Under these conditions, the specific rotation dropped rapidly at first and then more slowly until a constant value was reached after about 250 hours. In the two hundred strains studied, only two types of curve were found, and the proteins giving these were designated Protein I and Protein II.

Dr. B. N. Mitra undertook the study of possible structural differences in the two proteins, in respect to their amino acid arrangements. By isolating the amino acids from the two "racemized" proteins, and comparing their specific rotations with those of amino acids which had not been treated with alkali, he found certain differences. Thus, he concluded that glutamic acid and leucine differed completely in their relative positions in the two proteins, while arginine and proline were also somewhat differently placed.

Knowing that each strain of vibrios contained but one of the three polysaccharides and but one of the two proteins, it was possible to construct a chemical classification of the vibrios. The six groups are as follows:

Group	Protein Type	Polysaccharide Type
I	I	I
II	I	II
III	II	II
IV	II	I
V	II	III
VI	I	III

In addition to its interest from the point of view of chemical structure, the classification has also been a valuable guide in the study of the metabolism of the various types, and has permitted a study to be made of the chemical basis of variation. Finally it has enabled an answer to be given to the question with which the work started, that is, which vibrios are harmless and which dangerous. It appeared that organisms of the first two groups were the causative organisms of the disease, those of group III were harmless water vibrios, those of groups IV and V were found in the so-called chronic carriers of vibrios, who are probably without significance in the epidemiology of the disease, while group VI strains appeared to arise in old laboratory cultures.

The Glutamic Acid of Malignant Tissue Proteins.—GLADYS E. WOODWARD, FRANCIS E. REINHART AND JANETTA SCHOONOVER DOHAN. (*Journal of Biological Chemistry*, **138**: 677, 1941.) Amino acids isolated from hydrolysates of normal tissue proteins had been found, until recently, to belong to the same stereochemical series, and for this reason have been termed the "natural" or "l-amino acids." In 1939 Kögl and Erxleben (*Z. physiol. Chem.*, **258**: 57, 1939) were able to isolate from hydrolysates of malignant tissue proteins several amino acids containing varying amounts of the "unnatural" or d(—)-isomer, the amount in the case of glutamic acid being particularly high. Since they were able to isolate only the natural form from normal tissues they concluded that the presence of the unnatural isomer was a characteristic of malignancy.

The results of experiments in this laboratory on the isolation of glutamic acid were obtained from ten protein samples representing five malignant tumors (two rat, three human) and one non-malignant tumor (human). Two methods of isolation were used and were the same as those used by Kögl and Erxleben, namely (1) precipitation as the hydrochloride after cuprous oxide treatment of the hydrolysate, and (2) precipitation as the hydrochloride after extraction of the hydrolysate with butyl and propyl alcohol according to Dakin. In some cases residues were worked up in

order to obtain as much of the total glutamic acid as possible. The purity of all samples of glutamic acid hydrochloride was controlled by ash analyses and elementary analyses for C, H and N. In no case was a fraction of glutamic acid hydrochloride obtained which contained more than 4.4 per cent. of the d(—)-isomer; and the total d(—)-content of the sum of the samples isolated from any one tissue was in no case over 2.5 per cent. This was in contrast to amounts of 6 to 45 per cent. which had been found by Kögl and Erxleben.

Following Kögl's work, Chibnall (*Biochem. J.*, **34**: 285, 1940) had pointed out that when the percentages of d(—)-glutamic acid found by Kögl were calculated on the basis of the total weight of protein used, these were of the order of about 0.2 per cent. in 6 out of 9 cases, and that similar small amounts of the d(—)-isomer also had been isolated (by himself and others) from normal tissue proteins. Chibnall showed further that it was possible, at least under the conditions of the cuprous oxide method, to precipitate preferentially a small fraction of the total glutamic acid hydrochloride which contained all or most of the d(—)-form. The degree of racemization of an isolated sample could be dependent, therefore, on the yield, and since Kögl's yields were small his claim that the glutamic acid of tumors is highly racemized seems to be based largely on an incorrect interpretation of data.

In view of the importance of the extent of the yield, as indicated by Chibnall, it was desired in the present investigation to determine directly the amount of glutamic acid present in the hydrolysates. The method of Cohen (*Biochem. J.*, **33**: 551, 1939), which depends on conversion of glutamic to succinic acid and estimation of the latter by means of succinoxidase, was therefore adapted to protein hydrolysates. The hydrolysates, analyzed in this way in three cases, showed that the tumors contained 9.5 to 12.0 per cent. glutamic acid. In the remaining cases, where as much glutamic acid as possible had already been isolated, the analytical method was applied to the residues. Calculating on the basis of the sum of the glutamic acid isolated and that determined in the residues, values of 6.0 to 7.8 per cent. were obtained for the glutamic acid content of these tumors. These figures are

undoubtedly low since any acid that might have been lost during the isolation procedures had been neglected.

On the basis of the glutamic acid analyses by the Cohen method, the yield isolated was 47 to 70 per cent. of the total present in the tumors. If, as seems probable, the glutamic acid content of tumors lies in the range of 8 to 12 per cent., the amounts isolated by Kögl in the majority of cases represented only a small proportion (less than 10 per cent.) of the total present. In spite of the fact that the optical rotation of the glutamic acid isolated in these higher yields would be expected to be more representative of that of the total than in the low yields of Kögl, still these samples showed no appreciable racemization. The results therefore do not support the claim of Kögl that the glutamic acid of tumors is highly racemized.

The Sizes and Shapes of Protein Molecules by Ultracentrifugal Methods.—ELMER O. KRAEMER. (Abstract of lecture given in the Protein Symposium held under the auspices of the local section of the American Chemical Society at Princeton, New Jersey, and the Princeton Chemistry Department, March 6, 1941.)

After sketching the historical development of the Svedberg ultracentrifuges and the air-turbine ultracentrifuge of Beams, Bauer, Pickels and Wyckoff and pointing out the essential distinction between "ultracentrifuges" and various high-speed centrifuges such as the Sharples supercentrifuge, numerous medium and high-speed angle centrifuges and "quantity" centrifuges, a more detailed discussion of the molecular sizes and shapes of proteins was given.

Among the many outstanding discoveries made on proteins by Svedberg and his colleagues, the most significant are (1) the homogeneity in molecular weight of many of the well-defined proteins and (2) the simple multiple relations between the molecular weights of the homogeneous proteins. In both respects, these proteins are in a class by themselves among all the organic high molecular weight materials that have been studied. Because the general validity of these conclusions has recently been questioned by certain investigators, the degree of heterogeneity that may be detected with the ultra-

centrifuge and the exceptions to the multiple molecular weight rule were discussed.

Heterogeneity in molecular weight may be detected by means of sedimentation equilibrium and sedimentation velocity measurements, the latter being much the more sensitive indicator. For the former, heterogeneity is revealed by curvature in the plot of logarithm of concentration versus distance squared, measured from the axis of rotation, or by the steady increase in the molecular weights along the centrifuge cell, especially when calculated from values of the slope of the concentration gradient curve. The sensitivity of the sedimentation equilibrium to heterogeneity is substantially independent of molecular weight, within the range where a satisfactory equilibrium can be established and measured.

The better method for judging heterogeneity is by means of sedimentation velocity observations, if possible at such velocities that diffusivity is negligible. Such conditions can, however, rarely be attained with any except the very largest protein molecules. Usually the diffusivity is sufficiently great that molecular species differing by only a few per cent. in molecular weight are not distinctly separated. The possibility of detecting small degrees of heterogeneity then depends not only upon the resolving power of the ultracentrifuge, but also the sedimentation and diffusion constants of the molecules. For protein molecules with weights in excess of one million, Svedberg and his coworkers obtained clearly distinct separation of two components differing in molecular weight by less than 10 per cent. and it would accordingly have been possible to detect the presence of say ten per cent. of material differing by more than a few per cent. in molecular weight from the bulk of the material. As molecular weight decreases and diffusivity increases, sensitivity to heterogeneity decreases unless higher rates of sedimentation can be attained. Svedberg, *et al*, showed, for instance, that hemoglobin is homogeneous to a high degree by sedimentation at 900,000 times gravity. With the centrifugal forces usually employed (250,000–400,000 × gravity), the test for heterogeneity depends upon the comparison of the actual spreading of the sedimenting boundary and the spreading to be expected from the diffusion of a homogeneous material. Any discrepancy

is indicative of heterogeneity, but no quantitative specification of the heterogeneity can be made except on the basis of assumptions concerning the molecular weight distribution. Although relatively few calculations for effects of heterogeneity have been published, it would seem on the whole that ten per cent. contamination by molecular species differing more than ten per cent. from the bulk may be detected in proteins with molecular weights below 500,000.

The situation with regard to the rule of simple multiples is approximately the same as that for the periodic table of atomic weights, if isotopes are individually considered. Although there are exceptions to the rule, particularly among "fibrous" and other non-globular proteins, which are often more or less degraded during isolation, the number of cases fitting the rule is much greater than could arise from chance. Especially convincing are the results for the respiratory proteins, particularly those which were studied without isolation from their native fluids. A striking example of the small number of molecular sizes that occur is provided by a group of 22 hemocyanins from different species and their dissociation products (68 components), which fall into ten different well-marked groups. Some of the exceptions to the simple multiple rule are undoubtedly due to variations in the molecular weights of the amino acids making up the proteins, the rule probably depending actually on simple relations in the degree of polymerization of proteins. Other exceptions are probably due to the presence of non-amino acid components, such as carbohydrate, phosphatide, etc., attached to the protein molecule.

The shapes of molecules are more difficult to determine than their weights. Most methods involve the friction of the molecule moving either in a translational or rotational way in a solvent (e.g., in sedimentation, diffusion, viscosity, dielectric dispersion, etc.). The ratio of the observed friction to the friction that a spherical unsolvated molecule of the same molecular weight would show (the "frictional ratio") is then related by various theoretical equations with the axial ratios of equivalent ellipsoids. The frictional ratio is greater than unity for non-spherical molecules and for molecules whose volume is increased by combination with other components

of the solution. Fifty per cent. by weight hydration of a spherical protein molecule would give a frictional ratio of 1.2, which would correspond to an axial ratio of about 4.5. One hundred per cent. hydration would correspond to values of 1.33 and 6.8 respectively. Therefore, in the absence of independent evidence on hydration, frictional ratios do not lead to unambiguous information on particle shape if the shape is not definitely more asymmetrical than these figures. Of the 61 proteins for which Svedberg and Pedersen give molecular constants in their book, 42 have frictional ratios not exceeding 1.33, and 34 have ratios not definitely exceeding 1.2; therefore many of the well-defined proteins may well be substantially spherical, with more or less hydration.

On the other hand, other proteins which appear to be fibrous in character have considerably higher frictional ratios, and since the necessary hydration would be inconceivably large, the inference is reasonable that the molecules are elongated. This conclusion has been confirmed in certain cases by streaming birefringence observations, which provides a distinction between hydration and asymmetrical shape, at least when the asymmetry is sufficiently marked. Thus, gelatin has a frictional ratio of about 3.0 (the highest observed for a protein) which corresponds to an axial ratio of 37, ignoring possible hydration. The quantitative treatment of experimental data for such highly asymmetrical molecules at the present time is rather difficult and approximate in character for two reasons. Such elongated molecules have to be studied at very low concentrations in order to obtain satisfactory extrapolations of sedimentation and diffusion constant, intrinsic viscosity, etc., to zero concentration, and experimental difficulties often arise. In addition, such molecules are normally non-uniform in size, and methods for the calculation of significant or definable average values are just being worked out. However, a promising beginning has been made in investigating molecular shapes, and we may look forward to substantial increase in our knowledge along these lines when more use is made of streaming birefringence, and especially when the electron microscope is improved to the point of giving quantitative results for the many important proteins of moderate molecular size (e.g., $< 500{,}000$). Very interesting

results have already been obtained with the electron microscope for the very large molecules of tobacco mosaic virus, and there are German reports of photographs having been made on protein molecules as small as edestin, but it is questionable whether these are good enough to permit reliable calculations of molecular size. Some consideration will in addition have to be given to the effects on molecular size and shape of the extreme desiccation required for electron microscope observations.

BOOK REVIEWS.

EXPERIMENTAL ELECTRICAL ENGINEERING AND MANUAL FOR ELECTRICAL TESTING FOR ENGINEERS AND FOR STUDENTS IN ENGINEERING LABORATORIES, by V. Karapetoff. Vol. II, 4th Edition, 814 pages, illustrations, 16 × 24 cms. New York, John Wiley & Sons, Inc., 1941. Price $7.50.

This work has been well known to a host of engineers. First published in 1907 it went through editions of 1911, 1927 and now the fourth edition appears in revised and modernized form. The book at hand is volume two and carries on the study of certain subjects which were introduced in their simpler aspects in volume one, as well as additional material. It is a combination text and laboratory guide and as such, advantage is taken of the opportunity of having the student perform experiments at places in the text which will clarify and impress the subject matter. By following the text and suggested experiments it is easily seen that the procedure is a valuable one.

This volume picks up the thread where the first volume ended. It begins with chapter 26 on electrostatic capacitance and proceeds through a study of capacitance and inductance in A-C circuits. Eight experiments are outlined in this study. Then the text proceeds through the various topics concerned with power circuits and machinery. Voltage regulation on long and short transmission lines, fault location on lines including a description of the Allen and Gross fault locater, methods of measurement in symmetrical components, power measurements, transformations and conversions, synchronous machines, induction motors, single phase motors, and controllers for A.C. motors are all given expert treatment. Experiments illustrating methods and points of importance are injected at appropriate places in the text.

Attention is then given the oscillograph, to which a full chapter is devoted with explanations through experiments on the electromagnetic, hot-filament cathode ray, and Dufour type cathode ray oscillographs. Then there follows a discussion on harmonic analysis of periodic waves, a treatment on the fundamentals of electronic devices, and high frequency measurements.

The volume is a large one but nothing is spared in making the treatment complete. Simpleness of presentation without an excess of unimportant matter is outstanding which is plain evidence of much successful experience in teaching the subject. The many illustrations and diagrams, the comprehensive subject index, and the wealth of practical, usable material make this book a valuable one for study and reference.

R. H. OPPERMANN.

MODERN PRACTICE IN LEATHER MANUFACTURE, by John Arthur Wilson. 744 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1941. Price $9.50.

The leather industry from the ancients to the present has been a great industry. Through the ages it has met many needs of mankind and at times

leather has been most valuable. Today most of the leather manufactured comes from hides or skins of animals slaughtered for food. It is a by-product of the meat industry and the shoe manufacturer buys most of it. The leather industry is facing severe competition from substitutes developed by scientific manufacturers. In many uses leather has been displaced. The substitutes have also succeeded in finding new applications where leather might also be utilized. Even in shoes, substitutes are being used to an ever-increasing extent.

It is not at all unknown for an industry threatened with extinction to recognize its deficiencies and by taking them into account become alive again, develop further its product and uses for it, and take a new lease on life. The author of this book observes in the preface that although the operations of a tannery constitute a series of extremely complex chemical reactions and applications of physics equally complex, few practical tanners are versed in either chemistry or physics. This, in itself, is a serious charge particularly when considering the scientific mental equipment of competitors. In attempting to meet the very obvious need that exists the author presents here a treatment so written as to be understandable by anyone without the aid of a higher education. Every part of the leather business is treated on.

The histology of hides and skins is first discussed, followed by a description of how hides are prepared for the market with statistical reference to domestic production and imports and exports of hides and skins. A knowledge of the various types of damage common in hides and skins is next given and quite a thorough description is made of the hide and skin market, the purchase of the raw material and price fluctuations. A large portion of the book is devoted to tanning—handling of the raw stock, unhairing, bating, vegetable tanning, chrome tanning and miscellaneous tannages. The subject of finishing the leather after the tanning operation is taken up by giving attention to light leathers and their treatment from tanning to fatliquoring and mold control, then fatliquoring and dyeing them, drying and mechanical operations, and the procedure for various finishes. Sole and heavy leathers are taken up separately.

The last chapter of the book treats on the properties of leather. Here it is pointed out that all too often tanners take it for granted that the properties of leather are so well known that it is not necessary even to measure their values. This leaves an opening for competitors. In this chapter the properties, tests and apparatus for tests are thoroughly investigated.

The method of presentation, the logical divisions of the subject, the simpleness and completeness of treatment, the many illustrations, the broad scope, and the subject and name index make this book an unusual reliable source of information for all those interested in leather, no matter in what branch of the business that interest lies. Its use on a large scale undoubtedly would be reflected in the results of the leather industry.

R. H. OPPERMANN.

RADIOLOGY PHYSICS, by John Kellock Robertson. 270 pages, illustrations, plates, 16 × 24 cms. New York, D. Van Nostrand Company, Inc., 1941. Price $3.50.

This is a textbook on the elements of the subject designed primarily for medical, premedical and students of radiology, and represents the second and

concluding year's course by the author at Queen's University, Kingston, Canada. It is a book that is directed toward the generally accepted need for a knowledge in this field in view of the increasing applications of physics in medicine.

The treatment begins under the assumption that the student has had the usual course in general physics, a course covering a period of a year. To carry on from this point, electromagnetism is the first topic taken up going into alternating currents with their properties and characteristics. This leads to a description of the production of high voltage by means of the transformer, the rectification of alternating currents, the measurement of the currents, their control and conductivity through gases. On this basis there is discussed cathode rays and their properties, their uses in the generation of x-rays. The operation of Roentgen tubes is explained as well as the construction. A brief review of electromagnetic waves with particular reference to those of biological importance precedes a discussion of the subject in detail. This constitutes in general the first part of the book covering some 196 pages out of 245 pages of text.

Part two on the subject of the production of high voltage is inserted at this point, in the interest of logical progression of treatment. This section covers supervoltage production such as by the Van de Graaff generator. Considerable attention is given to the cyclotron. The topics of artificial radio-activity and high frequency currents complete the text. There is appended useful data on conversion factors, atomic numbers and weights and stable isotopes, a list of examination questions, and the international recommendations for x-ray and radium protection.

Throughout the treatment the author constantly recognizes the situation of the student, that the quantity of material must be limited, that the purpose is merely to give a working understanding of fundamentals, that simplicity is the keynote, that the knowledge imparted must be such as to be ready for practical application. The treatment is not hard to follow.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Mathematical Tables, by Herbert Bristol Dwight. 15 × 23 cms. McGraw-Hill Publishing Company, New York, 1941. Price $2.50.

Emulsions and Foams, Berkman and Egloff. 591 pages, illustrations. 15 × 23 cms. Reinhold Publishing Corporation, 1941. Price $8.50.

Fundamentals of Physical Science, by Konrad B. Krauskopf. 660 pages, illustrations, 16 × 23 cms. McGraw-Hill Book Company, Publishers, 1941. Price $3.50.

This Physical World, by C. C. Clark and C. A. Johnson. 528 pages, illustrations. 16 × 24 cms. McGraw-Hill Book Company, 1941. Price $3.25.

Education on an International Scale, by George W. Gray. 114 pages, illustrations. 15 × 22 cms. Harcourt, Brace & Co. 1941. Price $2.00.

CURRENT TOPICS.

Worthy of Note.—Many are the stories of success and achievement by men in all lines of endeavor. Generally the tenor of such stories is the same—a poor boy whose only qualifications included patience, tenacity, ambition, initiative, the proper attitude, an indomitable spirit, and others. A great deal has been said about the part that luck plays in success, and whether we like to or not, it must be admitted that fortunate breaks at times help to further a career. The story which has come to hand in this particular instance eliminates luck and definitely proves that a steady ambition and a fixed purpose plus hard work are the real masters of fate.

More than fifty years ago a young man sat in the auditorium of the Franklin Institue listening to a lecture. During the discourse the speaker happened to mention the fact that the skin markings of the human hand do not change during life time. The listener on the following morning inked the plam and fingers of his hand, made an imprint of them on a sheet of paper, and filed it away with a strange resolution to compare it with a new print fifty years later. This was done in 1887 when nothing was known in this country about fingerprint identification. In 1937 when comparison was made, it proved the fact. The incident is without precedent in the annals of fingerprint history.

This incident involved a young man who was not a student of identification. It illustrates a unique characteristic of the inquiring mind of Mr. W. N. Jennings, a member of many years standing of the Franklin Institute and the oldest living member of its Committee on Science and the Arts. Mr. Jennings' chosen field of endeavor was photography and it can truly be said that he is a pioneer in many phases. One of these was the photographing of lightning for which he was awarded the John Price Wetherill Medal by the Franklin Institute. This subject is the principal one in a scrap book compiled by Mr. Jennings over the years since 1882 and given by him to the Library of the Franklin Institute. That it is an interesting and valuable record is a foregone conclusion.

On the very first page Mr. Jennings has inserted a photograph of a lightning flash made by him in Philadelphia on September 2, 1882. This photograph, so far as he is aware, is the first of its kind ever made. It was first shown at the Franklin Institute on Sept. 5, 1882, when Mr. Jennings became a member. Subsequent photographs give a clear record of his early work on this subject. By means of original pamphlets, letters, etc., all included in the book it is revealed that the Royal Meteorological Society of London had

made a 12 month's investigation from which conclusions were drawn that lightning never occurs in the zig-zag form so commonly seen in pictures—a fact established long before by Mr. Jennings' photographs. The progress made by Mr. Jennings in photographing lightning is very striking as are the conclusions drawn from them. It showed definitely that the pictures were made the basis of much study. The book contains original letters from many of the foremost scientists of the day all in praise and appreciation of the work. They include such men as John Tyndall, Edison, Telsa, Thomson, Cleveland Abbe, Langley, Schuman and others. Recognition was also given by various journals and periodicals as is evidenced by clippings pasted in the book.

Early in Mr. Jennings' work, he became associated with Dr. Arthur W. Goodspeed of the University of Pennsylvania. They collaborated in many photoelectrical experiments, often done in the evening hours. It was while some of this work was being done that Mr. Jennings made the first photographic plate under an x-ray. This was three years before the discovery of x-rays by Dr. Roentgen. It was of a copper coin and was purely accidental. Mr. Jennings at the time failed to identify it.

Interest in other branches of photography by Mr. Jennings is shown by his association of many years in this country and abroad with F. E. Ives in the development of color photography. In 1893 his attention was attracted to balloon photography, and by employing orthochromatic plates and yellow color filters he succeeded in eliminating the effect of blue haze between balloon basket and landscape: This enabled clear sharp photographs to be taken from a balloon while passing over Philadelphia on a hazy afternoon. They were exhibited at the Franklin Institute in September, 1893. Another of Mr. Jennings' accomplishments was the design and construction of an eight by ten fixed-focus-multi-speed aero-camera with a between the lens shutter operating at 2000 per second. This was done with the advent of the airplane.

Mr. Jennings is a constant contributor to the JOURNAL OF THE FRANKLIN INSTITUTE and has lectured before that body many times. The book which he has so generously given to the Library has great value as a record of early achievement in the study and photography of electrical discharges. It is indeed a monument to a man who made use of the freedom and opportunities afforded him and contributed a wealth of good to the world. The Library of the Franklin Institute is fortunate in being custodian of this volume. The Franklin Institute is honored to have Mr. Jennings among its members and to have been a means of inspiration to him.

R. H. OPPERMANN.